Theorem The chi-square distribution is a special case of the Erlang distribution when $\alpha = 2$ and n is replaced with $n/2$, an integer.

Proof An Erlang random variable has probability density function

$$f(x) = \frac{1}{\alpha^n (n-1)!} x^{n-1} e^{-x/\alpha} \qquad x > 0.$$

When $\alpha = 2$ and $n = n/2$, this reduces to

$$\begin{aligned} f(x) &= \frac{1}{2^{n/2}(n/2-1)!} x^{n/2-1} e^{-x/2} \\ &= \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2} \qquad x > 0, \end{aligned}$$

which is the probability density function of the chi-square distribution with n degrees of freedom.

APPL verification: The APPL statements

```
assume(m, posint);
X := ErlangRV(1 / 2, m);
subs(m = n / 2, X[1][1]);
```

yield the probability density function of the chi-square distribution with minor regrouping when $m = n/2$ is an integer.