**Theorem** If $X$ is a $\chi(k)$ random variable then $Y = X^2$ is a $\chi^2(k)$ random variable.

**Proof** The cumulative distribution function of $Y$ is

$$\begin{aligned} F_Y(y) &= P(Y \le y) \\ &= P\left(X^2 \le y\right) \\ &= P\left(X \le y^{1/2}\right) \\ &= F_X\left(y^{1/2}\right) \qquad\qquad y > 0. \end{aligned}$$

Differentiating with respect to $y$ gives

$$\begin{aligned} f_Y(y) &= f_X(y^{1/2})\frac{1}{2y^{1/2}} \\ &= \frac{(1/2)^{k/2}}{\Gamma(k/2)} y^{k/2-1} e^{-y/2} \qquad\qquad y > 0, \end{aligned}$$

which is the probability density function of a $\chi^2(k)$ random variable.

**APPL Verification:** The APPL statements

```
X := ChiRV(k);
g := [[x -> x ^ 2, x -> x ^ 2], [-infinity, 0, infinity]];
Z := Transform(X, g);
ChiSquareRV(k);
```

confirm the result.