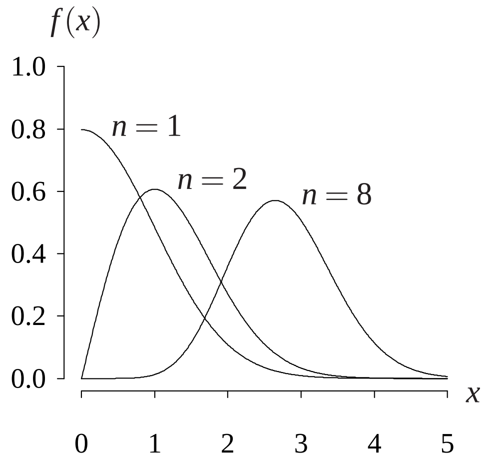
Chi distribution (from `http://www.math.wm.edu/~leemis/chart/UDR/UDR.html`)
The shorthand $X \sim \chi(n)$ is used to indicate that the random variable X has the χ (chi) distribution with n degrees of freedom, where n is a positive integer. A χ random variable X with n degrees of freedom has probability density function

$$f(x) = \frac{1}{2^{n/2-1}\Gamma(n/2)} x^{n-1} e^{-x^2/2} \qquad x > 0.$$

The probability density function, where $n = 1$, 2, and 8 is illustrated below.



The cumulative distribution function of X can be written as an integral that can't be evaluated in closed form. The expected value of X^k is

$$E\left[X^k\right] = \frac{2^{k/2}\Gamma(k/2+n/2)}{\Gamma(n/2)}$$

for $k = 1, 2, \ldots$. Using this result, the mean and variance are

$$E[X] = \frac{\sqrt{2}\,\Gamma(1/2+n/2)}{\Gamma(n/2)} \qquad V[X] = n - \mu^2$$

because

$$E\left[X^2\right] = \frac{2\Gamma(n/2+1)}{\Gamma(n/2)} = n.$$

The skewness and kurtosis are also written in terms of the gamma function. The mode is $\sqrt{n-1}$.

APPL verification: The APPL statements

```
X := ChiRV(n);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the population mean, variance, skewness, kurtosis, and moment generating function.