Theorem A zero-truncated Cauchy(a, γ) random variable is an arctangent(α, ϕ) random variable.

Proof The probability density function for a Cauchy random variable is

$$f(x) = \frac{1}{\gamma\pi[1 + ((x-a)/\gamma)^2]} \qquad -\infty < x < \infty.$$

The associated cumulative distribution function is

$$F(x) = \frac{1}{\pi}\arctan\left(\frac{x-a}{\gamma}\right) + \frac{1}{2} \qquad -\infty < x < \infty.$$

The probability density function of the zero-truncated random variable X is:

$$\begin{aligned}
\tilde{f}(x) &= \frac{f(x)}{1 - F(0)} \\
&= \frac{1}{\gamma\pi[1 + ((x-a)/\gamma)^2]\{1 - [\frac{1}{\pi}\arctan\left(\frac{0-a}{\gamma}\right) + \frac{1}{2}]\}} \\
&= \frac{1}{[\frac{1}{\pi}\arctan\left(\frac{a}{\gamma}\right) + \frac{1}{2}]\gamma\pi[1 + ((x-a)/\gamma)^2]} \\
&= \frac{1/\gamma}{[\arctan\left(\frac{a}{\gamma}\right) + \frac{\pi}{2}][1 + ((x-a)/\gamma)^2]} \qquad x > 0.
\end{aligned}$$

Setting $\alpha = 1/\gamma, \phi = a$,

$$\tilde{f}(x) = \frac{\alpha}{[\arctan(\alpha\phi) + \frac{\pi}{2}][1 + \alpha^2(x-\phi)^2]} \qquad x > 0,$$

which is the probability density function of the arctangent distribution.

APPL verification: The APPL statements

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X := CauchyRV(a, alpha);
Truncate(X, 0, infinity);
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yield the probability density function of the arctangent distribution indicated in the theorem.