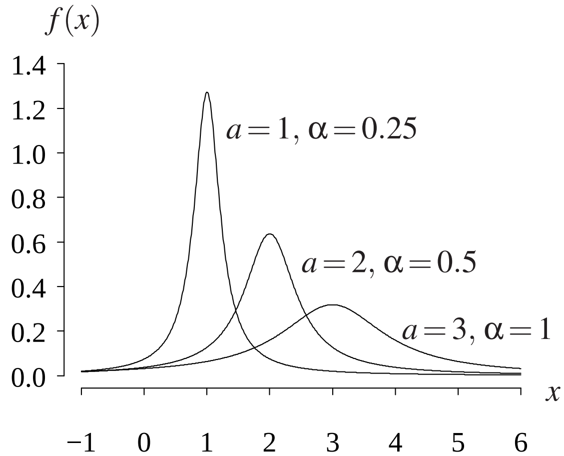
Cauchy distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{Cauchy}(a, \alpha)$ is used to indicate that the random variable X has the Cauchy distribution with positive scale parameter α and location parameter a. A Cauchy random variable X with parameters a and α has probability density function

$$f(x) = \frac{1}{\alpha\pi[1+((x-a)/\alpha)^2]} \qquad -\infty < x < \infty$$

for $\alpha > 0$ and $-\infty < a < \infty$. The Cauchy distribution is of interest because its moments are undefined. The probability density function for various combinations of a and α is given below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{1}{2\pi}\left(\pi - 2\arctan\left(\frac{a-x}{\alpha}\right)\right) \qquad -\infty < x < \infty.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{1}{2\pi}\left(\pi + 2\arctan\left(\frac{a-x}{\alpha}\right)\right) \qquad -\infty < x < \infty.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = 2\alpha\left(\alpha^2 - 2xa + a^2 + x^2\right)^{-1}\left(\pi + 2\arctan\left(\frac{a-x}{\alpha}\right)\right)^{-1} \qquad -\infty < x < \infty.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \ln(2\pi) - \ln\left(\pi + 2\arctan\left(\frac{a-x}{\alpha}\right)\right) \qquad -\infty < x < \infty.$$

The inverse distribution function of X is

$$F^{-1}(u) = a - \alpha\cot(\pi u) \qquad 0 < u < 1.$$

The population median of X is the location parameter, a.

The population moments of X are undefined. It follows that the population mean, variance, skewness, and kurtosis of X are also undefined.

APPL verification: The APPL statements

```
X := CauchyRV(a, alpha);
CDF(X);
SF(X);
HF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, mean, variance, skewness, kurtosis, and moment generating function.