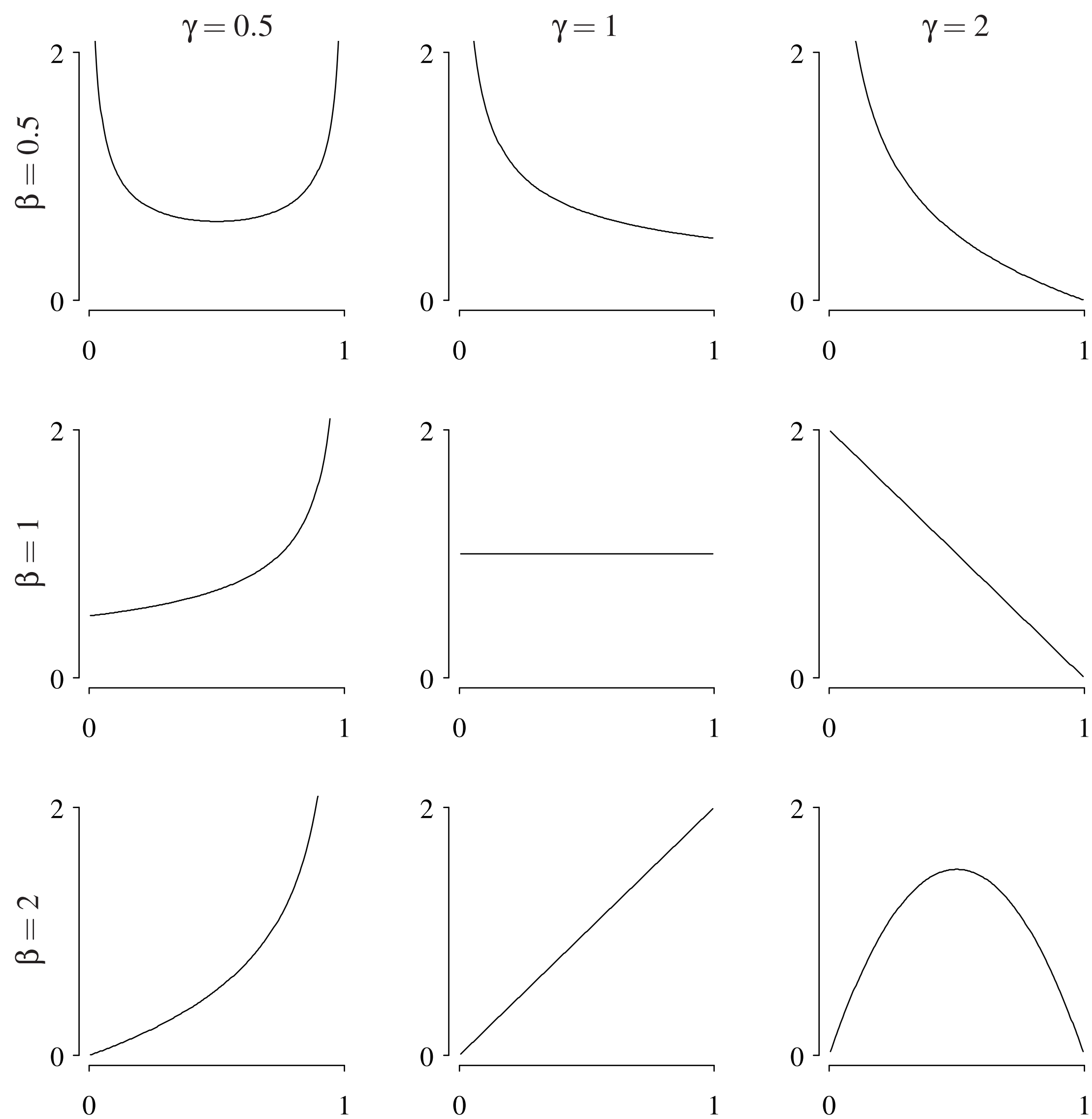
Beta distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{beta}(\beta, \gamma)$ is used to indicate that the random variable X has the beta distribution with parameters beta and gamma. A beta random variable X with positive shape parameters β and γ has probability density function

$$f(x) = \frac{\Gamma(\beta+\gamma)x^{\beta-1}(1-x)^{\gamma-1}}{\Gamma(\beta)\Gamma(\gamma)} \qquad 0 < x < 1.$$

The beta distribution is used for modeling random variables that lie between 0 and 1 (for example, percentages or interest rates) and as a prior distribution (for example, the beta–binomial distribution). Probability density functions with various values of the parameters are illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = I_x(\beta, \gamma) \qquad\qquad 0 < x < 1,$$

where I_x is the regularized incomplete beta function:

$$I_x(a,b) = \frac{B_x(a,b)}{B(a,b)},$$

where the beta function is

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and the incomplete beta function is

$$B_x(a,b) = \int_0^x t^{a-1}(1-t)^{b-1}\,dt.$$

For integer values of a and b, the regularized incomplete beta function can be computed via

$$I_x(a,b) = \sum_{j=a}^{a+b-1} \frac{(a+b-1)!}{j!(a+b-1-j)!} x^j (1-x)^{a+b-1-j}.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = 1 - I_x(\beta, \gamma) \qquad\qquad 0 < x < 1.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\Gamma(\beta+\gamma)x^{\beta-1}(1-x)^{\gamma-1}}{(1-I_x(\beta,\gamma))\Gamma(\beta)\Gamma(\gamma)} \qquad\qquad 0 < x < 1.$$

The cumulative hazard function and the inverse distribution function are mathematically intractable. The median of X is found by solving

$$I_m(\beta, \gamma) = 0.5$$

for m. The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{\beta}{\beta+\gamma}$$

$$V[X] = \frac{\beta\gamma}{(\beta+\gamma)^2(\beta+\gamma+1)}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{2(\gamma-\beta)\sqrt{1+\beta+\gamma}}{\sqrt{\beta\gamma}(\beta+\gamma+2)}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3\left(\beta^2\gamma+\beta\gamma^2-2\beta\gamma+2\beta^2+2\gamma^2\right)(1+\beta+\gamma)}{\beta\gamma(\beta+\gamma+2)(\beta+\gamma+3)}$$

APPL verification: The APPL statements

```
X := BetaRV(beta, g);
CDF(X);
SF(X);
HF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution function, survivor function, hazard function, population mean, variance, skewness, and kurtosis. Note the use of `g` as a parameter instead of `gamma` due to APPL error.