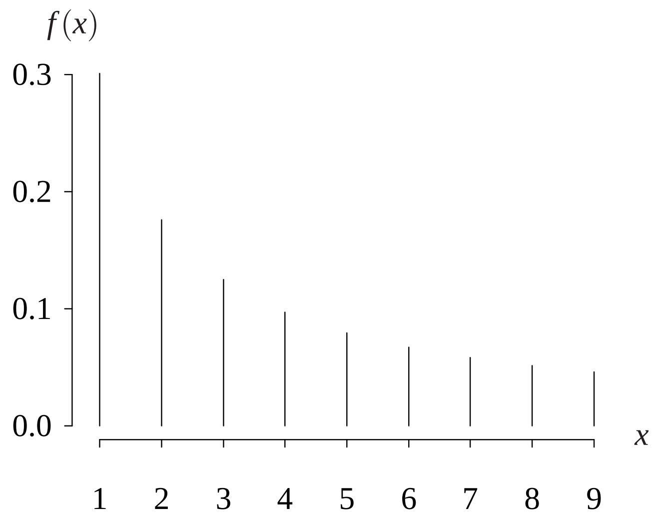
Benford distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim$ Benford is used to indicate that the random variable X has the Benford distribution. A Benford random variable X has probability mass function

$$f(x) = \log_{10}\left(1 + \frac{1}{x}\right) \qquad x = 1, 2, \ldots, 9.$$

The probability mass function is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \log_{10}(1 + x) \qquad x = 1, 2, \ldots, 9.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = 1 - \log_{10} x \qquad x = 1, 2, \ldots, 9.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\log_{10}\left(1 + \frac{1}{x}\right)}{1 - \log_{10} x} \qquad x = 1, 2, \ldots, 9.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = -\ln(1 - \log_{10} x) \qquad x = 1, 2, \ldots, 9.$$

The inverse distribution function is

$$F^{-1}(u) = \lfloor 10^u \rfloor \qquad 0 < u < 1.$$

The population mean, variance, skewness, and kurtosis of X is approximated with results from APPL

$$E[X] = \frac{2\ln(2) - 4\ln(3) + 8\ln(5) - \ln(7)}{\ln(2) + \ln(5)} \cong 3.4402$$

$$V[X] \cong 6.0565 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] \cong 0.7956 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] \cong 2.4518.$$

APPL verification: The APPL statements

```
X := BenfordRV();
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the population mean, variance, skewness, kurtosis and moment generating function.