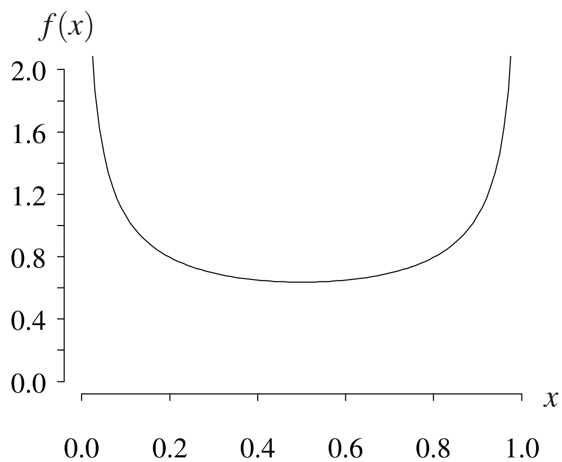
Arcsin distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim$ arcsin is used to indicate that the random variable X has the arcsin distribution. An arcsin random variable X has probability density function

$$f(x) = \frac{1}{\pi\sqrt{x(1-x)}} \qquad 0 < x < 1.$$

The probability density function is illustrated below.



The cumulative distribution on the support of X is

$$F(x) = P(X \le x) = \frac{\pi + 2\arcsin(2x-1)}{2\pi} \qquad 0 < x < 1.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{\pi - 2\arcsin(2x-1)}{2\pi} \qquad 0 < x < 1.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{2}{\sqrt{x(1-x)}(\pi - 2\arcsin(2x-1))} \qquad 0 < x < 1.$$

The inverse distribution function of X is

$$F^{-1}(u) = \frac{1}{2} - \frac{1}{2}\cos(\pi u) \qquad 0 < u < 1.$$

The cumulative hazard function, moment generating function, and characteristic function on the support of X are mathematically intractable.

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{1}{2} \qquad V[X] = \frac{1}{8} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3}{2}.$$

APPL verification: The APPL statements

```
X := [[x -> 1 / (Pi * sqrt(x * (1 - x)))], [0, 1], ["Continuous", "PDF"]];
CDF(X);
SF(X);
HF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, inverse distribution function, population mean, variance, skewness, kurtosis, and moment generating function.