Vol. 24 No. 2
MANAGEMENT ACCOUNTING Quarterly
Spring 2023

Budget Variance Analysis for Various Numbers and Combinations of Responsibility Centers

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EXECUTIVE SUMMARY

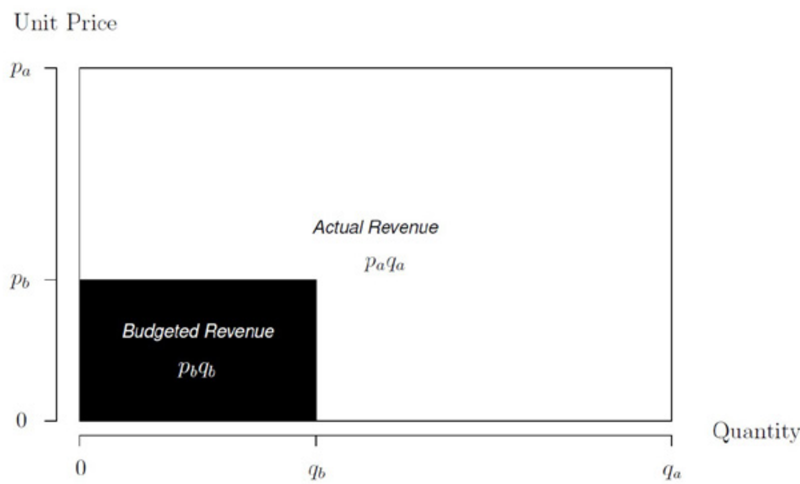
The most common variance analysis models are often used without consideration of whether or not they are appropriate for a given situation. We provide a set of models meant to be used when differentiation between situations based on number and combination of responsibility centers is important.

Studying the difference between budgeted (also known as "planned" or "standard") and actual numbers in a single period is a ubiquitous method in accounting and finance used in the variance analysis of various business performance metrics, such as cost, efficiency, productivity, profit, revenue, and so forth. Its application can be commonly found in disciplines such as marketing and operations management and in various industries such as air travel, construction, healthcare, and hospitality.[1]

While the definition of variance analysis is generally accepted, determining the appropriate criteria and frameworks to account for it has long provided challenges for academics and practitioners alike. As a result, the search for better alternatives to existing models continues, with justification such as "…the textbook example is solved by arbitrarily adding the joint variance to the price variance. There is no theoretical justification for so doing" and "…the conventional two-variance analysis (price and quantity) inflates variances in three of the four possible economic situations."[2]

There may never be a universally accepted "best" variance analysis model since there is "…no objective way of

Figure 1: General Model for Revenue Variance



generalizing which type of cost system was 'best' for control purposes" and "The plain fact is that competent managements use a variety of cost constructions for control purposes, and each may find merit in its own system."[3] So it is not surprising that undergraduate students may have difficulties grasping variance analysis of complex cost scenarios, let alone simple, common cases.[4]

Our goal here is to provide a variance analysis model that can be used in light of a company's strategic goals with respect to the number and combination of its responsibility centers.[5] This does not mean we are suggesting an ultimate, universal set of models, but one that we believe can be used when a primary goal is to evaluate the performance of accountable decision makers.

Consider two-variable revenue, in which the difference between budgeted and actual revenues is allocated to differences in unit price (price variance) and quantity sold (quantity variance). Figure 1 shows a general model for revenue variance, with budgeted price p_b, budgeted quantity sold (q_b), actual price (p_a), and actual quantity sold (q_a).

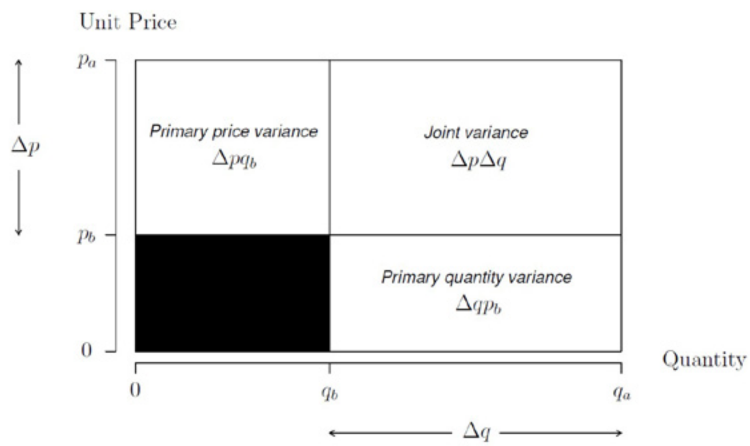
The area of the outer rectangle depicts actual revenue, the area of the inner (black) rectangle depicts budgeted revenue, and the difference (white area) depicts the revenue to be decomposed into price variance and quantity variance. Building from this, Figure 2 depicts the common starting point of the existing models in the literature. The difference of any variable x is calculated as $\Delta x = x_a - x_b$.

Existing models incorporate some form of joint variance similar to the depiction in Figure 2, with the debate over the past 100 years centering around its allocation. In mature or manufacturing settings, this joint variance may be relatively small compared to the primary variances. In growth and marketing settings, however, the joint variance (and impacts of its misallocation) can be significant.[6] Existing models are generated by allocating entire joint variance(s) to a single variable, often without justification. Assuming the general goal of variance analysis is to identify underlying causes of variation and help determine actionable steps for improvement, inaccurate variance models can provide misleading results.

The purpose of this article is to address issues that can arise when joint variance is misallocated, resulting in inaccurate variances. We do so by presenting an algorithm and certain resulting models that do not explicitly consider joint variance at all. Instead, we generate models in a logical fashion by incorporating the concept of responsibility centers and eliminating the need for arbitrary allocation of joint variance(s).

Through presentation of existing and proposed models with numerical examples, we demonstrate the consequences of using existing general models for

Figure 2: Intermediate Model for Revenue Variance



companies with different numbers and combinations of responsibility centers. While the proposed models are shown in the contexts of two-variable revenue, three-variable spending, and six-variable profit, the underlying mathematics can easily be applied to all aspects of variance analysis (as well as period-over-period horizontal analysis) with any number of associated variables. Our approach to generating variance analysis models is based on the concept of flexible budgeting, the use of which with variance analysis calculations is common.[7]

In general, prior studies and writings on variance analysis have focused on price and quantity using analytical, graphical, and applied works with regard to cost accounting. Early works defined price variance, quantity variance, joint variance, and net variance for two-variable models.[8] These definitions and calculations are relevant to manufacturing overhead calculations. Graphically, these variances are depicted similarly to Figure 2. In addition, prior works tend to stop at two-variable product models with little consideration of higher-level models, such as including a third variable (e.g., foreign exchange rate). Such higher-order variance analysis works that delve into models with more than three variables are rare.[9] We find this surprising, as any multinational manufacturing company would be converting foreign revenues and spending to domestic currency. This would be achieved by incorporating the exchange rate into two-variance revenue and three-variable direct costs, resulting in three- and four-variable products, respectively.

MODEL GENERATION

Our original models are based on two generally held premises of budget variance analysis. First, "…it is important to designate the portion of the total variance that is attributable to each manager."[10] In other words, models should be a function of the number and combination of responsibility centers. To account for the possibility that for an n-variable product, there may be any of 0, 1, 2, … , n responsibility centers, $n + 1$ general-form models (i.e., one for zero responsibility centers, one for one responsibility center, etc.) can be generated. More specifically, $2n$ models can be generated accounting for all possible combinations of n variables, each of which is or is not attributed to a responsibility center.

As we will demonstrate in the next sections, the zero- and n-responsibility center models are the same. Therefore, for an n-variable product, there are $2n - 1$ unique, final models. We are unaware of any existing models derived explicitly from this premise.

Second, a flexible budget is an estimate of what revenue and costs should have been had managers known the actual value of variables not assigned to their respective responsibility centers in advance. Robert Kaplan, a leading expert on the topic of variance analysis, states, "…managers expect that many of their indirect and support expenses should be managed or controlled based on actual activity levels during the period."[11] We refer to this as "Kaplan's premise." Given this, our models are generated such that the variance of a variable that is attributed to a responsibility center will be calculated using the actual value of the variable(s) not attributed to a responsibility center. A corollary is that the variance of a variable that is not attributed to a responsibility center will be calculated using the budgeted value of the variable(s) that is (are) attributed to a responsibility center.

Note that there may be cases where the latter premise is debatable, such as demand being a function of price. It may not be reasonable to expect a sales department to meet its respective budget of units sold if that selling price increases. That said, we adopt Kaplan's premise and acknowledge circumstances under which it may not be appropriate.

In generating the most commonly used revenue variance model (discussed in the next section, "Two-Variable Product Variances"), quantity variance is calculated using the budgeted value of price. The quantity variable is then "flexed" from its budgeted to actual value to calculate price variance. Considering Kaplan's premise, this suggests that there is a responsibility center for setting unit price, but not one for sales. Thus, with a two-variable product, there is one flexing step. It is easy to show that for an n-variable product there will be $n - 1$ flexing steps. In generating our models, we follow the general procedure of calculating a variance, flexing the respective variable, calculating another variance, flexing the respective variable, etc., until all variances have been calculated.

Note that for a two-variable product with one responsibility center, there will be only one model that satisfies the above requirements. For all other cases of $n \geq 2$, there will be multiple possible models resulting from the different orders in which the variances are calculated and variables are flexed. In these cases, the final model is calculated as the average of all the possible applicable models. This addresses an outstanding issue related to the order in which variables are flexed to generate a model.[12] We assume no preference for the order in which variables are flexed, other than all those not assigned to a responsibility center are flexed before all those that are.

The algorithm used for generating our models (the Algorithm) is presented in the appendix and demonstrated with a three-variable, one-responsibility center example. Results for the specific cases of two-variable revenue, three-variable spending, and five-variable profit are in the following sections of this article. The original algorithm; the general results for two, three and four-variable products; and a numerical example are available online.[13]

TWO-VARIABLE PRODUCT VARIANCES

The two-variable case is presented in the context of revenue variance, incorporating unit price p and quantity sold q. Subscripts a and b denote actual and budgeted values, respectively. The difference of any variable x is calculated as $\Delta x = x_a - x_b$. The average of any variable x is calculated as $\bar{x} = (x_a + x_b) / 2$.

Where applicable, the company's marketing department is responsible for determining unit price, while the sales department is responsible for quantity sold. Such a setting would occur in practice when management would like to retrospectively assign how much of the difference in revenue is associated with a difference in unit price and how much is associated with a difference in quantity sold. This partitioning would help management assess the relative impacts of the marketing and sales departments with respect to difference in revenue. Using the Algorithm, we generate the $2^2 - 1 = 3$ unique, final models used for the two-variable revenue case and present them in Table 1. A numerical example and a discussion follow.

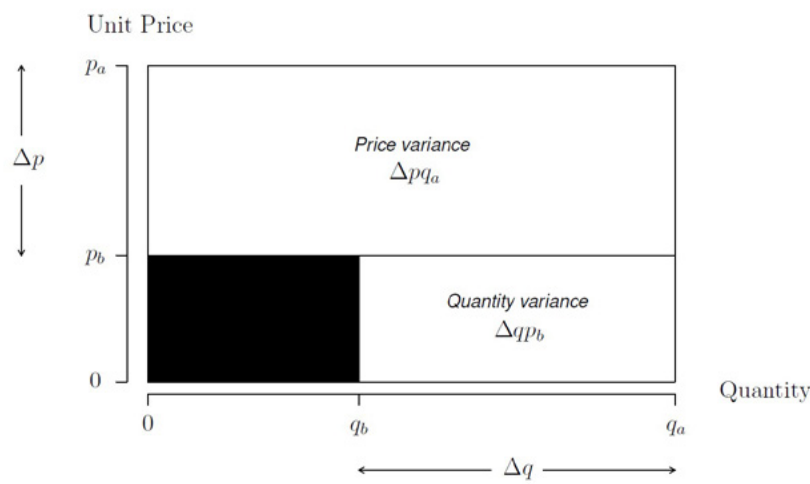
Model 1A (Figure 3) is the same as what is most commonly found in the research literature and introductory textbooks. It is equivalent to allocating the entire joint variance to price variance. Given our two premises, we propose this as a reasonable model to use when the marketing department is the only responsibility center. An example would be a company with pricing power but where the number of units sold is determined by the market, given the marketing department's choice of price.

Table 1: Variance Models for Two-Variable Revenue

Variance	Responsibility Center(s): Marketing 1A	Sales 1B	Marketing and Sales* 1C
Price	$\Delta p q_a$	$\Delta p q_b$	$\Delta p \bar{q}$
Quantity	$\Delta q p_b$	$\Delta q p_a$	$\Delta q \bar{p}$
Revenue	$p_a q_a - p_b q_b$		

*Also applicable for the case of no responsibility centers.

Figure 3: Revenue Variance for Marketing Responsibility Center



As depicted in Figure 4, Model 1B is equivalent to a model found by allocating the entire joint variance to quantity variance. We propose this as a reasonable model to use when the sales department is the only responsibility center. An example would be a company selling a pure commodity and with no pricing power. Such a company can decide how much supply to make available for sale (or more simply, only meet exact demand that has been determined with preorders), assuming that all that is supplied will be sold at the spot price.

In addition to being final models for the cases previously discussed, Models 1A and 1B are the two intermediate models used to generate Model 1C (see Figure 5). We propose this as a reasonable model to use when both the sales and marketing departments are responsibility centers. An example would be a company that belongs to a cartel and has pricing power as well as the ability to decide how much supply to make available.

Interestingly, Model 1C can also be appropriate for when there are no responsibility centers. An example would be a company selling a commodity where neither the selling price nor the supply is under the direct control of the company. Consider a company that owns solar panels and sells all collected energy to an energy grid for distribution to end users. The spot price is set by either the market or the distributor, and the amount of sunlight available to be collected is determined by weather and time of day.

Figure 4: Revenue Variance for Sales Responsibility Center

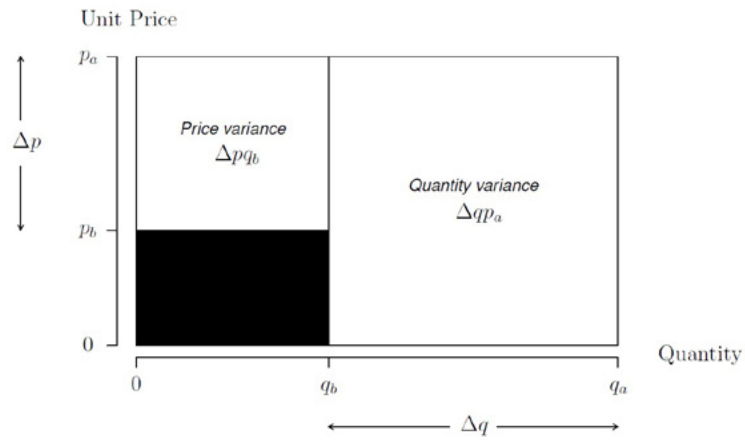
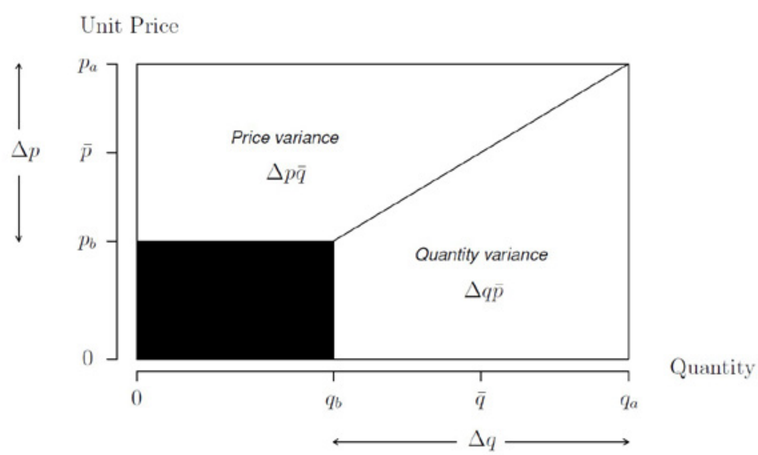


Figure 5: Revenue Variance for Marketing and Sales Responsibility Centers



A benefit of using variance analysis for such a scenario where none of the variables can be influenced by a decision maker is to determine if it may be economical to create a responsibility center for one (or both) of the variables in the future. If analysis reveals a significant price variance, for example, it may be worthwhile to hire an expert to negotiate pricing contracts rather than be at the mercy of the market.

Example 1. For discussion purposes, we provide a numerical example using the parameters in Table 2. Note that Case 2 is the mirror image of Case 1—simply that the budgeted and actual parameters are switched.

Using the models in Table 1, we obtain the results shown in Table 3. The labels in Table 3 correspond to the models in Table 1.

Consider an interpretation of the results using Model 1C. In Case 1, the \$1,200 revenue variance can be broken down as \$150 due to the difference in unit price and \$1,050 due to the difference in quantity sold. Stated in another fashion, one-eighth of the increase in revenue (\$150 / \$1,200) can be attributed to

Table 2: Parameters for Two-Variable Revenue Example

Variable	Case 1 Budgeted	Case 1 Actual	Case 2 Budgeted	Case 2 Actual
Unit Price	$10	$11	$11	$10
Quantity	100	200	200	100

Table 3: Results of Two-Variable Revenue Example

Variance	Case 1 Responsibility Center(s)		
	Marketing 1A	Sales 1B	Marketing and Sales* 1C
Price	$200	$100	$150
Quantity	$1,000	$1,100	$1,050
Revenue	$1,200		

Variance	Case 2 Responsibility Center(s)		
	Marketing 1A	Sales 1B	Marketing and Sales* 1C
Price	($100)	($200)	($150)
Quantity	($1,100)	($1,000)	($1,050)
Revenue	($1,200)		

*Also applicable for the case of no responsibility centers.

the difference in unit price, and seven-eighths of the increase in revenue ($1,050 / $1,200) can be attributed to the difference in the quantity sold.

Note the nontrivial differences in each variance resulting from using the single responsibility center models (1A and 1B). Using Case 1 as an example, consider the price variances of $200 and $100. The difference demonstrates the issue that can arise when multiple models are available to calculate a variance. Assuming that Model 1B is an appropriate model to use when the sales department is the only responsibility center, using Model 1A results in a 100% increase from the appropriate price variance. This specific issue is common in practice since Model 1A is generally adopted to be used for cases of two-variable revenue variance analysis. Similar issues may arise when a company is using a single-responsibility center model but could be using Model 1C in the cases where neither or both the marketing and sales departments are responsibility centers.

Also, compare the values of a variable's variance between the two cases. Given the cases are mirror images, we might intuitively expect that each Case 2 variance would be of equal magnitude, but opposite sign, of its respective Case 1 variance. Using Model 1A as an example, we might expect the price variance for Case 2, ($100), to be the additive inverse of the price variance for Case 1, $200, but this is not the case. This property is seen with Model 1C, but with neither Model 1A nor 1B. This can be explained analytically and relies on Kaplan's premise. Should this premise not be applicable for a company, a case can be made that, because of the asymmetrical forms of Models 1A and 1B, it might be appropriate for all cases of two-variable variance analysis for that particular company to use Model 1C.

In regard to the general form of the two single-responsibility center models using Model 1A, note that each of q_a and p_b appears in both variances as either part of a difference or as itself. Yet q_b only appears in the quantity variance as part of Δq, and p_a only appears in the price variance as part of Δp. The implications of this are that a change in q_a or p_b results in a change in both component variances, but a change in q_b or p_a results in a change in only one component variance. An analogous pattern is seen with Model 1B. Conversely, with Model 1C a change in any one of p_a, p_b, q_a, and q_b results in a change in both component variances. These characteristics also result from adopting Kaplan's premise. Again, should this not be applicable for a company, or if the company wants a change in any of the variables' values to be reflected by a change in all variances, there is justification that Model 1C may be used for all cases of two-variable product variance analysis.

Finally, note that the decision of which model to use may be based on desired emphasis of one variance over the other. For example, Model 1A would be used to emphasize price variance as it shows both greater contribution in the case of increased revenue and at the same time lesser detraction in the case of decreased revenue. By similar reasoning, Model 1B would be used to emphasize quantity variance.

The implications for evaluating performance, determining executive compensation, or valuing a company are obvious. Anyone standing to benefit from the appearance of improved pricing will use Model 1A, while those who benefit from the appearance of improved sales will prefer the results from Model 1B. Conversely, Model 1C does not have a predetermined emphasis toward either variance. Its selection may be based on the desire for gaining neutral, unbiased insight, and/or when Kaplan's premise is not applicable.

THREE-VARIABLE PRODUCT VARIANCES

We present the three-variable case as a spending example that would appear in a direct materials purchases budget of a company. This includes unit cost or c (price per unit of input paid by the company), usage or u (units of input used per unit of output), and quantity or q (units of output produced and assumed to be sold). Subscripts a and b denote actual and budgeted values, respectively. The difference of any variable x is calculated as $\Delta x = x_a - x_b$. The average of any variable x is calculated as $\bar{x} = (x_a + x_b) / 2$.

Where the responsibility centers exist, the purchasing department is responsible for unit cost, the production department is responsible for usage, and the sales department is responsible for quantity. For this example, we will show how to generate all possible models for the case of three-variable spending variance analysis.

To begin, there are six ($n! = 3! = 6$) possible intermediate models for a three-variable product, generated using the Algorithm. These are found by assuming three (or zero) responsibility centers and performing steps 1 through 9 (see Table 4).

Table 5 lists the intermediate variance models in Table 4 used in the generation of the seven ($2^3 - 1 = 7$) unique, final models for all possible numbers and combinations of responsibility centers. For example, to generate the variance analysis model for the case of the production department being the only responsibility center, using the Algorithm will result in first generating Intermediate Models 4C and 4D. Note that in both models, (1) usage variance is calculated using only the actual value of unit cost and quantity, and (2) unit cost and quantity variances are calculated using only the budgeted value of usage. The Algorithm then requires taking the average of these two intermediate models, yielding Model 6B. That and all other unique final models are shown in Table 6.

Table 4: Intermediate Variance Models for Spending

Variance	4A	4B	4C	4D	4E	4F
Unit Cost	$\Delta c q_a u_a$	$\Delta c q_a u_a$	$\Delta c q_b u_b$	$\Delta c q_a u_b$	$\Delta c q_b u_a$	$\Delta c q_b u_b$
Usage	$\Delta u c_b q_a$	$\Delta u c_b q_b$	$\Delta u c_a q_a$	$\Delta u c_a q_a$	$\Delta u c_b q_b$	$\Delta u c_a q_b$
Quantity	$\Delta q c_b u_b$	$\Delta q c_b u_a$	$\Delta q c_a u_b$	$\Delta q c_b u_b$	$\Delta q c_a u_a$	$\Delta q c_a u_a$

Table 5: Intermediate Variance Models Included for Final Model Determination

Responsibility Center(s)		
Purchasing	Production	Sales
4A and 4B	4C and 4D	4E and 4F

Responsibility Center(s)			
Purchasing and Production	Purchasing and Sales	Production and Sales	Purchasing, Production, and Sales*
4A and 4D	4B and 4E	4C and 4F	All

*Also applicable for the case of no responsibility centers.

Example 2. Next, let us look at a numerical example. Table 7 contains the parameters. Note that Case 2 is the mirror image of Case 1—the budgeted and actual parameters are switched. Using the models in Table 6, we obtain the results shown in Table 8. The labels in Table 8 correspond to the models in Table 6.

The same insights from the two-variable example can be gained from the three-variable example. First, there can be nontrivial differences between calculated and true variances if multiple models are used. Second, given the asymmetrical forms of Models 6A-6F, a case can be made for using the *n*-responsibility center model (Model 6G) for all cases of three-variable variance analysis. Third, the only model for which changing the value of one variable (either actual or budgeted) changes the variance of all variables is the *n*-responsibility center model (Model 6G). Finally, any of Models 6A-6F can be chosen with the goal of emphasizing one or more variances over another, but the *n*-responsibility center model (6G) does not have a predetermined bias toward any variance.

The three-variable example leads to a new insight: If the actual value of a variable x equals its budgeted value ($x_a = x_b = x$), the three-variable model reduces to its respective two-variable model, scaled by x. Consider the following examples using Model 6B, a three-variable, one-responsibility center (the purchasing department) model.

1. If $c_a = c_b = c$, then unit cost variance = 0, usage variance = $\Delta u q_a c$, and quantity variance = $\Delta q u_b c$. This is a two-variable, one-responsibility center model of the same form as Models 1A and 1B, scaled by c.

2. If $u_a = u_b = u$, then unit cost variance = $\Delta c \bar{q} u$, usage variance = 0, and quantity variance = $\Delta q \bar{c} u$. This is a two-variable, zero-responsibility center model of the same form as Model 1C, scaled by c.

Table 6: Variance Models for Three-Variable Spending

Variance	Responsibility Center(s) Purchasing 6A	Production 6B	Sales 6C
Unit Cost	$\Delta c q_a u_a$	$\Delta c \bar{q} u_b$	$\Delta c q_b \bar{u}$
Usage	$\Delta u c_b \bar{q}$	$\Delta u c_a q_a$	$\Delta u \bar{c} q_b$
Quantity	$\Delta q c_b \bar{u}$	$\Delta q \bar{c} u_b$	$\Delta q c_a u_a$
Spending	$c_a u_a q_a - c_b u_b q_b$		

Variance	Responsibility Center(s) Purchasing and Production 6D	Purchasing and Sales 6E	Production and Sales 6F	Purchasing, Production, and Sales* 6G
Unit Cost	$\Delta c q_a \bar{u}$	$\Delta c \bar{q} u_a$	$\Delta c q_b u_b$	$\Delta c \left(\bar{q}\bar{u} + \frac{\Delta q \Delta u}{12} \right)$
Usage	$\Delta u \bar{c} q_a$	$\Delta u c_b q_b$	$\Delta u c_a \bar{q}$	$\Delta u \left(\bar{c}\bar{q} + \frac{\Delta c \Delta q}{12} \right)$
Quantity	$\Delta q c_b u_b$	$\Delta q \bar{c} u_a$	$\Delta q c_a \bar{u}$	$\Delta q \left(\bar{c}\bar{u} + \frac{\Delta c \Delta u}{12} \right)$
Spending	$c_a u_a q_a - c_b u_b q_b$			

*Also applicable for the case of no responsibility centers.

Table 7: Parameters for Three-Variable Spending Example

Variable	Case 1 Budgeted	Case 1 Actual	Case 2 Budgeted	Case 2 Actual
Unit Cost	\$5	\$6	\$6	\$5
Usage	1.40	1.52	1.52	1.40
Quantity	100	200	200	100

3. If $q_a = q_b = q$, then unit cost variance = $\Delta c u_b q$, usage variance = $\Delta u c_a q$, and quantity variance = 0. This is a two-variable, one-responsibility center model of the same form as Models 1A and 1B, scaled by q.

The underlying mathematics of this three-variable spending example are applicable to any three-variable product case. Another application would be revenue, by incorporating exchange rate on unit price in the example presented in the "Two-Variable Product Variances" section. Similarly, the spending case in this section could be expanded to a four-variable case by incorporating exchange rate on unit cost. Both these cases would be of obvious interest to a multinational corporation.

Table 8: Results of Three-Variable Spending Example

Variance	Purchasing	Production	Sales	Purchasing and Production	Purchasing and Sales	Production and Sales	Purchasing, Production, and Sales*
	Case 1						
	Responsibility Center(s)						
	6A	6B	6C	6D	6E	6F	6G
Unit Cost	$304	$210	$146	$292	$228	$140	$220
Usage	$90	$144	$66	$132	$60	$108	$100
Quantity	$730	$770	$912	$700	$836	$876	$804
Spending	$1,124						

Variance	Purchasing	Production	Sales	Purchasing and Production	Purchasing and Sales	Production and Sales	Purchasing, Production, and Sales*
	Case 2						
	Responsibility Center(s)						
	6A	6B	6C	6D	6E	6F	6G
Unit Cost	($140)	($228)	($292)	($146)	($210)	($304)	($220)
Usage	($108)	($60)	($132)	($66)	($144)	($90)	($100)
Quantity	($876)	($836)	($700)	($912)	($770)	($730)	($804)
Spending	($1,124)						

*Also applicable for the case of no responsibility centers.

PROFIT VARIANCES

To demonstrate the additive properties of the models with a profit function, we will use the two-variable revenue case, the three-variable spending case, and another, new three-variable spending case that would appear in a direct labor cost budget of a company.

For the new case, we use labor rate or l (price of labor per unit of production time), production time or t (units of production time required per unit of output), and quantity or q (units of output produced). Subscripts a and b denote actual and budgeted values, respectively. Without loss of generality, we omit factory variable overhead for the sake of brevity. We assume that all units produced are also sold, meaning the quantity used for expense calculations is the same as the quantity used for revenue calculations. The result is the profit function $\pi(c, l, p, q, t, u) = pq - (cuq + ltq)$. We use the n-responsibility center models for demonstration purposes. Subtracting Model 6G and the analogous model for direct labor cost from Model 1C yields the variance model in Table 9.

Consider the following generalizations based on that example: For products, a change in any of the component variables' actual or budgeted value results in a change in all the component variables' variances. For sums, this is not the case. For example, a change in actual unit cost results in a change in both usage and quantity variances, but not in any variances of unit price, labor rate, or production time.

A change in actual or budgeted quantity results in a change of all variances only because it appears in every product in the profit function. Likewise, a change in actual or budgeted value of any of the other five variables will result in a change in quantity variance.

With the negative sign on the unit cost, usage, labor rate, and production time variances, a positive

Table 9: Variance Model for Profit with *n* Responsibility Centers

Variable	Variance
Unit Price	$\Delta p\bar{q}$
Unit Cost	$-\Delta c\left(\bar{q}\bar{u}+\dfrac{\Delta q\Delta u}{12}\right)$
Usage	$-\Delta u\left(\bar{c}\bar{q}+\dfrac{\Delta c\Delta q}{12}\right)$
Labor Rate	$-\Delta l\left(\bar{q}\bar{t}+\dfrac{\Delta q\Delta t}{12}\right)$
Production Time	$-\Delta t\left(\bar{l}\bar{q}+\dfrac{\Delta l\Delta q}{12}\right)$
Quantity	$\Delta q\left(\bar{p}-\bar{c}\bar{u}-\dfrac{\Delta c\Delta u}{12}-\bar{l}\bar{t}-\dfrac{\Delta l\Delta t}{12}\right)$
Profit	$p_aq_a-(c_au_aq_a+l_at_aq_a)-(p_bq_b-(c_bu_bq_b+l_bt_bq_b))$

difference (i.e., actual value is greater than budgeted value) results in a negative variance. This is logically consistent with the concept of an unfavorable result, as would be the case with a negative difference (i.e., actual value is less than budgeted value) in unit price also resulting in a negative variance.[14]

GENERATING *n*-VARIABLE MODELS

Companies are using budget variance analysis models that are often based on the arbitrary allocation of joint variance. We showed the negative implications of companies with different numbers and/or combinations of responsibility centers using existing variance analysis models. To overcome this, we used an algorithm (the Algorithm) to be used to generate *n*-variable models for all possible numbers and combinations of responsibility centers in the companies. The models were presented in the context of budgeted vs. actual amounts in a single period, with the underlying mathematics easily applied to other forms of analysis such as period-over-period comparisons (horizontal analysis). ■

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ENDNOTES

1 For marketing examples, see Ted Mitchell and Howard Olsen, "Marketing Control: Exogenous Aspects of Price Variance," *Journal of Business & Economics Research*, December 2003, pp. 23-32; Ted Mitchell and Mike Thomas, "Can Variance Analysis Make Media Marketing Managers More Accountable?" *Management Accounting Quarterly*, Fall 2005, pp. 51-61; Ted J. Mitchell and Mike Thomas, "The Need for a New Accounting Variance Model to Evaluate Sales Performance," *International Handbook of Academic Research and Teaching*, 2009, pp. 2-11. For operations management examples, see Nicholas Dopuch, "A Perspective on Cost Drivers," *The Accounting Review*, July 1993, pp. 615-620; Harper A. Roehm, Larry Weinstein, and Joseph F. Castellano, "Management Control Systems: How SPC Enhances Budgeting and Standard Costing," *Management Accounting Quarterly*, Fall 2000, pp. 34-40; Thomas J. Wilke, "The Project Cost Variance Analysis

Model: A Project Management Tool," *The Journal of Cost Analysis & Management*, November 2005, pp. 13-23; Michael F. Thomas and James Mackey, "Supply Chain Management: Monitoring Strategic Partnering Contracts with Activity-Based Measures," *Management Accounting Quarterly*, Fall 2006, pp. 1-10; David Bukovinsky and John C. Talbott, "Variance Analysis Using Throughput Accounting," *The CPA Journal*, January 2010, pp. 28-35; Manjunath H.S. Rao and Andrew Bargerstock, "Exploring the Role of Standard Costing in Lean Manufacturing Enterprises: A Structuration Theory Approach," *Management Accounting Quarterly*, Fall 2011, pp. 47-60; Kenneth Snead, David Stott, and Andy Garcia, "You Should Have Seen that Volume Variance Coming!" *Management Accounting Quarterly*, Fall 2013, pp. 11-20; Lawrence P. Grasso and Thomas Tyson, "Management Accounting Systems and Performance Measurement at Lean Companies," *Management Accounting Quarterly*, Summer 2021, pp. 1-14. For air travel, see Paul A. Mudde and Parvez E. Sopariwala, "Examining Southwest Airlines' Strategic Execution: A Strategic Variance Analysis," *Management Accounting Quarterly*, Summer 2008, pp. 20-32. For construction, see Jarosław Gorecki and Manuel Diaz-Madronero, "Who Risks and Wins?—Simulated Cost Variance in Sustainable Construction Projects," *Sustainability*, April 2020, pp. 1-31. For healthcare, see Ge Bai, "Applying Variance Analysis to Understand California Hospitals' Expense Recovery Status by Patient Groups," *Accounting Horizons*, June 2016, pp. 211-223. For hospitality, see Ken Milani and Aaron Perri, "Managing Meal Costs: Variance Generation, Analysis, and Interpretation, *Management Accounting Quarterly*, Summer 2013, pp. 1-11.

2 Josef Kloock and Ulf Schiller, "Marginal Costing: Cost Budgeting and Cost Variance Analysis," *Management Accounting Research*, September 1997, pp. 299-323; Mitchell and Thomas, 2005.

3 Robert N. Anthony, "Cost Concepts for Control," *The Accounting Review*, April 1957, pp. 229–234.

4 Kennard T. Wing, "Using Enhanced Cost Models in Variance Analysis for Better Control and Decision Making," *Management Accounting Quarterly*, Winter 2000, pp. 1-9; Neal R. VanZante, "Helping Students See the 'Big Picture' of Variance Analysis," *Management Accounting Quarterly*, Spring 2007, pp. 39-47.

5 Vijay Govindarajan and John K. Shank, "Profit Variance Analysis: A Strategic Focus," *Issues in Accounting Education*, Fall 1989, pp. 396-410.

6 Mitchell and Thomas, 2005.

7 David W. Young, "Note on Flexible Budgeting and Variance Analysis," Harvard Business School Publishing, March 13, 2013.

8 H.W. Maynard, "The Accounting Technique for Standard Costs," *National Association of Cost Accountants Bulletin*, 1927, pp. 542-563; Gilbert Amerman, "The Mathematics of Variance Analysis – Part I," *Accounting Research*, 1953, pp. 258-269; Gilbert Amerman, "The Mathematics of Variance Analysis – Part II," *Accounting Research*, 1953, pp. 329-350; Gilbert Amerman, "The Mathematics of Variance Analysis – Part III," *Accounting Research*, 1954, pp. 56-79; K.C. Banerjee, "The Mathematics of Variance Analysis and the Possibilities of Its Application," *The Accounting Review*, 1953, pp. 351-363; Ching-Wen Kwang and Albert Slavin, "The Simple Mathematics of Variance Analysis," *The Accounting Review*, July 1962, pp. 415-432; Charles Weber, "The Mathematics of Variance Analysis," *The Accounting Review*, July 1963, pp. 534-539.

9 C.Y. Tang and H. Davis, "Analyzing Activity Cost Variances," *Advances in Management Accounting*, 2000, pp. 171-188; Thomas and Mitchell, 2009; Peter Almström, "Productivity Measurement and Improvements: A Theoretical Model and Applications from the Manufacturing Industry," IFIP International Conference on Advances in Production Management Systems, pp. 297-304; Craig M. Sorochuk, C. Ryan Vinroot, and Lawrence M. Leemis, "Budget Variance Analysis of *n*-Variable Products with Zero or *n* Responsibility Centers," *Journal of Theoretical Accounting Research*, 2023, pp. 1-14.

10 Young, 2013.

11 Robert S. Kaplan, "Variance Analysis and Flexible Budgeting," Harvard Business School Background Note 101-039, October 2000.

12 Tang and Davis, 2000.

13 They are available at scholarship.shu.edu/comp-decision-sciences-faculty-pubs/1/.

14 Sia Nassiripour and Lianzan Xu, "A New Method Towards Better Comprehension of Variance Analysis," *Journal of Accounting & Finance Research*, Winter 2004, pp. 34-40.

Appendix: Algorithm for Generating Models

Generally, the Algorithm begins with all variables set to their budgeted values. It progressively alters each variable (those of nonresponsibility centers first) to its actual value once the respective variance has been calculated.

We will demonstrate the Algorithm using a three-variable product $w(x, y, z) = xyz$, where x and y are variables that are not attributed to a responsibility center and z is a variable that is attributed to a responsibility center. Each variable can take on two values, actual and budgeted (denoted by subscripts a and b, respectively). The difference of any variable x is calculated as $\Delta x = x_a - x_b$. The average of any variable x is calculated as $\bar{x} = (x_a + x_b) / 2$. (The Algorithm is illustrated by the bulleted items.)

1. Begin a new intermediate model with all variables taking on their respective budgeted values:

- x_b, y_b, and zb

2. Select a variable that is not attributed to a responsibility center. If there are none, select a remaining variable:

- Select x

3. Calculate the variance for the selected variable as its difference multiplied by the product of the budgeted values of the remaining $n - 1$ variables:

- Calculate x variance as $(x_a - x_b)y_b z_b = \Delta x y_b z_b$

4. Update the value of the selected variable to its actual value, to be used in subsequent calculations:

- x_b is updated to x_a while y_b and z_b remain unchanged

5. Select a variable that is not attributed to a responsibility center whose variance has not yet been calculated. If there are none, select a remaining variable for which a variance has not yet been calculated:

- Select y

6. For this newly elected variable, calculate its variance as its difference multiplied by the product of the values of the remaining $n - 1$ variables:

- Calculate y variance as $(y_a - y_b)x_a z_b = \Delta y x_a z_b$

7. Update the value of the newly selected variable to its actual value, to be used in subsequent calculations:

- y_b is updated to y_a while x_a and z_b remain unchanged

8. Repeat steps 5 through 7 until a variance has been calculated for each variable. This set of variances comprises the given intermediate model:

- Step 5 (second pass): select z
- Step 6 (second pass): z variance = $\Delta z x_a y_a$
- Step 7 (second pass): z_b is updated to z_a while x_a and y_a remain unchanged
- Intermediate Model A: x variance = $\Delta x y_b z_b$, y variance = $\Delta y x_a z_b$, and z variance = $\Delta z x_a y_a$

9. Repeat steps 1 through 8 until all intermediate models have been generated:

- Step 1: x_b, y_b, and z_b
- Step 2: Select y
- Step 3: Calculate y variance as $(y_a - y_b)x_b x_b = \Delta y x_b z_b$
- Step 4: y_b is updated to y_a while x_b and z_b remain unchanged
- Step 5: Select x
- Step 6: x variance = $\Delta x y_a z_b$
- Step 7: x_b is updated to x_a while y_a and z_b remain unchanged
- Step 8:
    - o Step 5 (second pass): Select z

- o Step 6 (second pass): z variance = $\Delta z x_a y_a$
- o Step 7 (second pass): z_b is updated to z_a while x_a and y_a remain unchanged
- o Intermediate Model B: x variance = $\Delta x y_b z_b$, y variance = $\Delta y x_a z_b$, and z variance = $\Delta z x_a y_a$

Table A1

Variable	Intermediate Model	
	A	B
x	$\Delta x y_b z_b$	$\Delta x y_a z_b$
y	$\Delta y x_a z_b$	$\Delta y x_b z_b$
z	$\Delta z x_a y_a$	$\Delta z x_a y_a$

10. For each variable, calculate its variance as the average of its respective variances from the intermediate models. The set of the variances comprises the final model.

Table A2

Variable	Model
x	$\frac{\Delta x y_b z_b + \Delta x y_a z_b}{2} = \Delta x z_b \frac{y_b + y_a}{2} = \Delta x \bar{y} z_b$
y	$\frac{\Delta y x_a z_b + \Delta y x_b z_b}{2} = \Delta y z_b \frac{x_a + x_b}{2} = \Delta y \bar{x} z_b$
z	$\frac{\Delta z x_a y_a + \Delta z x_a y_a}{2} = \Delta z \frac{2 x_a y_a}{2} = \Delta z x_a y_a$