Appendix: Algorithm

Procedure `KS`: Computing the CDF of the Kolmogorov–Smirnov test statistic

`Input`: A positive integer n, the sample size associated with the KS test statistic when all parameters are known.
`Output`: The CDF of the KS test statistic when all parameters are known is returned. The random variable will be in the usual list-of-lists format.

[**Phase 1:** Determine the endpoints of the segments that define the support of the KS test statistic.]

Dimension $v\left[0..\left\lceil\frac{3n}{2}\right\rceil-1\right]$ [v holds the partition values for the support of D_n]
$m \leftarrow 0$
$v_m \leftarrow 0$
$g \leftarrow \frac{1}{2n}$ [g is the gap between adjacent integration limits when $v=0$]
For $i \leftarrow 1$ to $n-1$ by 1
    $m \leftarrow m+1$
    $v_m \leftarrow i \cdot g$
For $j \leftarrow 2 \cdot \lfloor\frac{n}{2}\rfloor + 1$ to $2n-1$ by 2 [Lower limit is n or $n+1$, whichever is odd]
    $m \leftarrow m+1$
    $v_m \leftarrow j \cdot g$

[**Phase 2:** The next phase of the algorithm has three parts to it.

1. Define $c_1, c_2, \ldots, c_m$ as the midpoints of the support intervals
2. Define $x_1, x_2, \ldots, x_n$ as the limits of integration when $v=0$
3. Define the $n \times n$ indicator matrices $A_1, A_2, \ldots, A_m$.]

Dimension $c[m]$ [Define vector c of midpoints of the support subintervals]
For $k \leftarrow 1$ to m
    $c_k \leftarrow (v_{k-1}+v_k)/2$

Dimension $x[n]$ [Define vector x of centers of integration limits]
For $i \leftarrow 1$ to n
    $x_i \leftarrow (2i-1) \cdot g$

Dimension $A[n,n]$

For $i \leftarrow 2$ to n
    For $j \leftarrow 1$ to $i-1$
        $A_{ij} \leftarrow 0$ [Zero out lower triangle of A]

For $k \leftarrow 1$ to m [Loop through all subintervals of support]
  For $i \leftarrow 1$ to n
    For $j \leftarrow i$ to n
      $A_{ij} \leftarrow 0$ [Zero out upper triangle of A]
  $z \leftarrow \max\{\lfloor nc_k - \frac{1}{2} \rfloor, 0\}$ [z is the number of leading zero columns in A]
  $l \leftarrow \min\{\lceil 2nc_k \rceil, n\}$ [l is the number of u-subintervals that intersect $\left(\frac{1}{2n} - v, \frac{1}{2n} + v\right)$]
  For $i \leftarrow 1$ to n
    For $j \leftarrow \max\{i, z+1\}$ to $\min\{n, i+l-1\}$
      $A_{ij} \leftarrow 1$ [Place 1's in A]

[**Phase 3:** Compute the fixed and variable limit matrices F and V and combine to give the piecewise polynomial CDF]

Dimension $P[m]$ [nth order polynomials for CDF segments, parameterized by v]
Dimension $F[n, n]$ [Array of Fixed integrals, parameterized by v]
Dimension $V[n, n]$ [Array of Variable integrals, parameterized by v]
Real $S(v)$ [A function of v, used as a running sum]
  [$S(v)$ is the sum of all F elements one row down, starting one element to the right]

$P_1(v) \leftarrow n!(2v)^n$ $\left[P_1(v) \leftarrow n! \int_{x_1-v}^{x_1+v} \int_{x_2-v}^{x_2+v} \ldots \int_{x_n-v}^{x_n+v} 1 \, du_n \, \ldots \, du_2 \, du_1\right]$
For $k \leftarrow 2$ to m [Loop through all subintervals of support]
  $z \leftarrow \max\{\lfloor nc_k - \frac{1}{2} \rfloor, 0\}$
  $l \leftarrow \min\{\lceil 2nc_k \rceil, n\}$
  $F_{n,n}(v) \leftarrow \int_{x_n-v}^{1} 1 \, du_n$
  $V_{n,n}(v) \leftarrow \int_{u_{n-1}}^{1} 1 \, du_n$
  For $i \leftarrow n-1$ to 2 by -1
    If $i + l > n$ then
      $S(v) \leftarrow 0$
    else
      $S(v) \leftarrow F_{i+1,i+l}(v)$
    If $i + l > n + 1$ then
      $F_{i,n}(v) \leftarrow \int_{x_n-v}^{\lfloor x_i + c_k \rfloor + (x_i+v)(1-\lfloor x_i + c_k \rfloor)} V_{i+1,n}(v) \, du_i$
      $V_{i,n}(v) \leftarrow \int_{u_{i-1}}^{\lfloor x_i + c_k \rfloor + (x_i+v)(1-\lfloor x_i + c_k \rfloor)} V_{i+1,n}(v) \, du_i$
    If $i + l = n + 1$ then
      $F_{i,n}(v) \leftarrow \int_{x_n-v}^{x_i+v} V_{i+1,n}(v) \, du_i$
    If $i + l < n + 1$ then
      $F_{i,i+l-1}(v) \leftarrow \int_{x_{i+l-1}-v}^{x_i+v} \{V_{i+1,i+l-1}(v) + S(v)\} \, du_i$
    $S(v) \leftarrow S(v) + F_{i+1,\min\{i+l-1,n\}}(v)$
    For $j \leftarrow \min\{n-1, i+l-2\}$ to $\max\{i+1, z+2\}$ by -1 [Interior of A matrix]
      $F_{i,j}(v) \leftarrow \int_{x_j-v}^{x_{j+1}-v} \{V_{i+1,j}(v) + S(v)\} \, du_i$
      $V_{i,j}(v) \leftarrow \int_{u_{i-1}}^{x_{j+1}-v} \{V_{i+1,j}(v) + S(v)\} \, du_i$
      $S(v) \leftarrow S(v) + F_{i+1,j}(v)$
    If $z + 1 \le i$ then
      $V_{i,i}(v) \leftarrow \int_{u_{i-1}}^{x_{i+1}-v} S(v) \, du_i$

If $z+1>i$ then
$V_{i,z+1}(v) \leftarrow \int_{u_{i-1}}^{x_{z+2}-v} \{V_{i+1,z+1}(v)+S(v)\}\, du_i$
If $z+1<i$ then
$F_{i,i}(v) \leftarrow \int_{x_i-v}^{x_{i+1}-v} S(v)\, du_i$
If $l=n$ then [Rows $2,3,\ldots,n$ of A matrix completed at this point]
$S(v) \leftarrow 0$
$F_{1,n}(v) \leftarrow \int_{x_n-v}^{x_1+v} V_{2,n}(v)\, du_1$
else
$S(v) \leftarrow F_{2,l+1}(v)$
If $l<n$ then
$F_{1,l}(v) \leftarrow \int_{x_l-v}^{x_1+v} \{V_{2,l}(v)+S(v)\}\, du_1$
$S(v) \leftarrow S(v)+F_{2,l}(v)$
For $j \leftarrow \min\{n-1,l-1\}$ to $\max\{2,z+1\}$ by -1
$F_{1,j}(v) \leftarrow \int_{(x_j-v)\lceil x_j-c_k\rceil}^{x_{j+1}-v} \{V_{2,j}(v)+S(v)\}\, du_1$
$S(v) \leftarrow S(v)+F_{2,j}(v)$
If $z=0$ then
$F_{1,1}(v) \leftarrow \int_0^{x_2-v} S(v)\, du_1$
$P_k(v) \leftarrow 0$
For $j \leftarrow z+1$ to l
$P_k(v) \leftarrow P_k(v)+F_{1,j}(v)$
$P_k(v) \leftarrow n! P_k(v)$

[**Phase 4:** Shift the distribution to the form of $P(D_n<y)$ by substituting $y=v+\frac{1}{2n}$.]

$y_0 \leftarrow v_0+\frac{1}{2n}$
For $k \leftarrow 1$ to m [Loop through all subintervals of support]
$y_i \leftarrow v_k+\frac{1}{2n}$
$F_k(y) \leftarrow P_k(y-\frac{1}{2n})$

`Return:` $y_0,y_1,\ldots,y_m$ and $F_1(y),F_2(y),\ldots,F_m(y)$.