The Counterfeit Coin Problems

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More generally, if there are k coins with $3^{m-1} < k \leq 3^m$, then we need only m weighings.

k:	$1-3$	$4-9$	$10-27$	$28-81$	$82-243$
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If there is a p pan balance, then one can find the fake coin out of k coins by m weighings if $(p+1)^{m-1} < k \leq (p+1)^m$.

A More Difficult Problem

Suppose 12 coins are given such that one of them has a different weight. Use three weighings to find the different coin, and determine whether it is heavier or lighter.

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More challenging problems

* How many weighings to find a different coin from k given coins.

* What if there are two lighter / different coins?

* What if there are three lighter / different coins?