Homework #4 Part A

Write the expressions in Problems 1 through 4 as a single power of $b$, where the value of $b$ is stated in each problem. No explanations are needed on 1-5, just show all of your steps.

1. $4^{1/4} \cdot \left(\frac{1}{2}\right)^{-3}$, with $b = 2$.

2. $\frac{9^{1/6} \cdot 3^{1/2}}{3^{-3} \cdot 3^4}$, with $b = 3$.

3. $(1000)^{1/3} \cdot 10^7 \cdot (10^2)^3 \cdot (100)^{-1/2}$, with $b = 10$.

4. $\sqrt[4]{b^4} \cdot (b^2)^{3/5} \cdot b^{-3}$, where $b$ might be any nonzero real number. What happens if $b = 0$?

5. Write the following as a single power of $x^2 + 1$:

$$\left(\frac{\sqrt[3]{(x^2 + 1)^2}}{\sqrt{(x^2 + 1)^3} \cdot (x^2 + 1)^{1/3}}\right)^2$$

6. Find all values of $x$ for which the following expression is defined:

$$(x^2 - x - 6)^{1/4}.$$

7. Find all values of $x$ such that $x - 2x^{1/2} - 8 = 0$

Hint: This is a quadratic in $x^{1/2}$, so if $z = x^{1/2}$, then $z^2 = x$, and the equation becomes $z^2 - 2z - 8 = 0$. But also remember that $x^{1/2} = \sqrt{x}$ means the positive square root, so $\sqrt{x} \geq 0$ for all $x \geq 0$.

8. For each of the following, decide whether each statement is True or False. Give a one sentence explanation and/or a clear computation with your answer:

(a): The expression $\sqrt{x - 2}$ is defined for every possible real number $x$.

(b): The expression on the left-hand side below can be simplified as follows:

$$\sqrt{[(x^{1/2} - x^{-1/2})^2 + 4]} = x^{1/2} + x^{-1/2}.$$