Problem Set 1

Math 213, Fall 2016

Name:

P1. Determine the projection of \( \mathbf{a} \) onto \( \mathbf{b} \).

a) \( \mathbf{a} = (2, 1, -3), \quad \mathbf{b} = (-1, 1, 1) \)

b) \( \mathbf{a} = (-1, 1, 1), \quad \mathbf{b} = (2, 1, -3) \)

P2. Let \( \mathbf{a} = (a_1, a_2, a_3) \) be an arbitrary vector in \( \mathbb{R}^3 \) and let \( \varphi, \theta, \gamma \) denote the angle between \( \mathbf{a} \) and the \( x, y, z \) axes, respectively. Show that

\[
\cos^2 \varphi + \cos^2 \theta + \cos^2 \gamma = 1.
\]

P3. Determine the area of the parallelogram generated by \( \mathbf{a} \) and \( \mathbf{b} \).

a) \( \mathbf{a} = (1, -2, 1), \quad \mathbf{b} = (2, -1, 2) \)

b) \( \mathbf{a} = (3, 4, 5), \quad \mathbf{b} = (1, 1, -1) \)

c) \( \mathbf{a} = (3, 1, -1), \quad \mathbf{b} = (-6, -2, -2) \)

P4. Compute the determinant of the matrix

\[
A = \begin{pmatrix}
1 & 1 & 1 \\
2 & 3 & 4 \\
4 & 9 & 16
\end{pmatrix}.
\]

P5. Determine the volume of the parallelepiped generated by \( \mathbf{a}, \mathbf{b} \) and \( \mathbf{c} \).

a) \( \mathbf{a} = (2, 1, -1), \quad \mathbf{b} = (5, 0, -3), \quad \mathbf{c} = (1, -2, 1) \)

b) \( \mathbf{a} = (1, 0, 0), \quad \mathbf{b} = (0, 3, -1), \quad \mathbf{c} = (4, 2, -1) \)

P6.

a) Determine an equation for the plane orthogonal to \( (1, 1, 1) \) through the point with coordinates \( (1, 0, 0) \).

b) Determine an equation for the plane orthogonal to \( (1, -2, -1) \) through the point with coordinates \( (3, 4, 5) \).

P7. Determine the intersection of the planes \( x + 2y + z = 0 \) and \( x - 3y - z = 0 \).