Problem 1. Consider the linear system $Ax = 0$, where

$$A = \begin{pmatrix} 3 & 1 & -2 \\ 2 & 0 & -2 \\ 4 & 0 & -4 \end{pmatrix}.$$ 

a) Determine whether the homogeneous linear system $Ax = 0$ possesses a nontrivial solution and describe the solution-set in parametric vector form.

b) If the columns of $A$ are denoted by $v_1, v_2, v_3$, is the collection of vectors $\{v_1, v_2, v_3\}$ linearly independent in $\mathbb{R}^3$?

c) Is the collection $\{v_1, v_3\}$ linearly independent? Why?

Problem 2.

a) Is the set $\{v_1, v_2, v_3, v_4\}$, where $v_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ -1 \\ -5 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ -5 \\ -16 \end{pmatrix}$, and $v_4 = \begin{pmatrix} 2 \\ 5 \\ -19 \\ 0 \end{pmatrix}$ linearly independent in $\mathbb{R}^4$? If not, provide a linear dependence relation.
b) Is the set \(\{v_1, v_2\}\) as in part (a) linearly independent? Why?

**Problem 3.** Suppose \(A\) is a \(3 \times 3\) matrix that is row-equivalent to

\[
U = \begin{bmatrix}
\blacksquare & \star & \star \\
0 & \blacksquare & \star \\
0 & 0 & \blacksquare 
\end{bmatrix},
\]

where \(\blacksquare\) is nonzero and \(\star\) is real. Please answer and provide adequate justification for the following questions:

a) Does \(Ax = 0\) possess a nontrivial solution?

b) Are the columns of \(A\) linearly independent?

c) Do the columns of \(A\) span \(\mathbb{R}^3\)?