Problem for Homework #7

Math 430 - Spring 2017

(a): Let R be a ring with 1, and let I be a proper ideal of R. Prove that if R has no maximal ideal containing I, then the ring R/I has no maximal ideal. In particular, conclude that if there exist rings with 1 with proper ideals contained in no maximal ideal, then there exist rings with 1 which have no maximal ideals.

(b): Suppose that R is a ring with 1 which has no maximal ideal, and suppose that F is any field. Consider the ring $R \times F$. Prove that $M = R \times \{0\}$ is the unique maximal ideal of $R \times F$ (so $R \times F$ is a local ring), but that there are non-units which are in $R \times F$ but not in M.
Hint: Use the fact that we saw on an earlier HW about the structure of any ideal of a direct product of two rings.