Homework #5 Problems

Math 430 - Spring 2013

1. Let $R = \mathbb{Z}\left[\frac{1+\sqrt{-19}}{2}\right] = \{a + b\left(\frac{1+\sqrt{-19}}{2}\right) \mid a, b \in \mathbb{Z}\}$.
(a): Prove that R is a subring of $\mathbb{C}$, and conclude that R is an integral domain.
(b): Define $N : R \to \mathbb{Z}$ by

$$N\left(a + b(1+\sqrt{-19})/2\right) = a^2 + ab + 5b^2 = (a + b/2)^2 + (19/4)b^2.$$

Prove that N is a multiplicative norm on R.
(c): Find all units in R.

2. Let D be an integral domain. A *Dedekind-Hasse norm* on D is a function $N : D \to \mathbb{Z}_{\geq 0}$ such that $N(a) = 0$ if and only if $a = 0$, and given any nonzero $a, b \in D$, either $b|a$ in D, or there exist $s, t \in D$ such that $0 < N(sa - tb) < N(b)$. Prove that if D is an integral domain on which there exists a Dedekind-Hasse norm, then N is a PID. (Hint: Use the same type of idea used to show that any Euclidean domain is a PID).