1. Let \( R = \mathbb{Z} \left[ \frac{1+\sqrt{-19}}{2} \right] = \{ a + b \left( \frac{1+\sqrt{-19}}{2} \right) \mid a, b \in \mathbb{Z} \} \).
(a): Prove that \( R \) is a subring of \( \mathbb{C} \), and conclude that \( R \) is an integral domain.
(b): Define \( N : R \to \mathbb{Z} \) by
\[
N \left( a + b \left( \frac{1+\sqrt{-19}}{2} \right) \right) = a^2 + ab + 5b^2 = \left( a + \frac{b}{2} \right)^2 + \frac{19}{4} b^2.
\]
Prove that \( N \) is a multiplicative norm on \( R \).
(c): Find all units in \( R \).

2. Let \( D \) be an integral domain. A Dedekind-Hasse norm on \( D \) is a function \( N : D \to \mathbb{Z}_{\geq 0} \) such that \( N(a) = 0 \) if and only if \( a = 0 \), and given any nonzero \( a, b \in D \), either \( b \mid a \) in \( D \), or there exist \( s, t \in D \) such that \( 0 < N(sa - tb) < N(b) \). Prove that if \( D \) is an integral domain on which there exists a Dedekind-Hasse norm, then \( N \) is a PID. (Hint: Use the same type of idea used to show that any Euclidean domain is a PID).