Homework #1 Problems

Math 430 - Spring 2013

1. Suppose $G$ is a finite abelian group. Prove that any composition series for $G$, \( \{e\} = H_0 \triangleleft H_1 \triangleleft \cdots \triangleleft H_{m-1} \triangleleft H_m = G \), is such that $H_i/H_{i-1}$ is cyclic of prime order for $i = 1, 2, \ldots, m$.
   (Hint: Since $G$ is abelian, each $H_i$ must be abelian, and we know each $H_i/H_{i-1}$ is simple.)

2. Suppose $K$ and $H$ are groups such that $K \triangleleft H$ and $H/K$ is a finite abelian group. Prove that there are subgroups $K_i$ such that $K = K_0 \triangleleft K_1 \triangleleft \cdots \triangleleft K_{m-1} \triangleleft K_m = H$ such that $K_i/K_{i-1}$ is cyclic of prime order for $i = 1, 2, \ldots, m$.
   (Hint: First apply Problem 1 to the group $H/K$, then pull back to subgroups of $H$.)

3. Use the above two problems to prove the following. Suppose that $G$ is a finite group which has a subnormal series \( \{e\} = N_0 \triangleleft N_1 \triangleleft \cdots \triangleleft N_{m-1} \triangleleft N_m = G \) such that $N_i/N_{i-1}$ is abelian for $i = 1, 2, \ldots, m$. Prove that $G$ has a composition series \( \{e\} = H_0 \triangleleft H_1 \triangleleft \cdots \triangleleft H_{\ell-1} \triangleleft H_\ell = G \) such that $H_i/H_{i-1}$ is cyclic of prime order for $i = 1, 2, \ldots, \ell$. 