Homework #1 Problems

Math 430 - Spring 2013

1. Suppose G is a finite abelian group. Prove that any composition series for G, $\{e\} = H_0 \lhd H_1 \lhd \cdots \lhd H_{m-1} \lhd H_m = G$, is such that H_i/H_{i-1} is cyclic of prime order for $i = 1, 2, \ldots, m$.
(Hint: Since G is abelian, each H_i must be abelian, and we know each H_i/H_{i-1} is simple.)

2. Suppose K and H are groups such that $K \lhd H$ and H/K is a finite abelian group. Prove that there are subgroups K_i such that $K = K_0 \lhd K_1 \lhd \cdots \lhd K_{m-1} \lhd K_m = H$ such that K_i/K_{i-1} is cyclic of prime order for $i = 1, 2, \ldots, m$.
(Hint: First apply Problem **1** to the group H/K, then pull back to subgroups of H.)

3. Use the above two problems to prove the following. Suppose that G is a finite group which has a subnormal series $\{e\} = N_0 \lhd N_1 \lhd \cdots \lhd N_{m-1} \lhd N_m = G$ such that N_i/N_{i-1} is abelian for $i = 1, 2, \ldots, m$. Prove that G has a composition series $\{e\} = H_0 \lhd H_1 \lhd \cdots \lhd H_{\ell-1} \lhd H_\ell = G$ such that H_i/H_{i-1} is cyclic of prime order for $i = 1, 2, \ldots, \ell$.