Problem for Homework #4

Math 430 - Spring 2011

In class, we have shown that if D is an integral domain which has a Dedekind-Hasse norm, then D is a PID. For this problem, you will show the converse statement, finishing off the proof that an integral domain D is a PID if and only if there exists a Dedekind-Hasse norm on D.

Let D be a PID. Define a function $N : D \to \mathbb{Z}_{\geq 0}$ as follows. Let $N(0) = 0$, $N(u) = 1$ for any unit $u \in D^{\times}$, and for any nonzero non-unit element $a \in D$, write a as a product of irreducible elements of D, say $a = p_1 \cdots p_n$, and define $N(a) = 2^n$. Prove that N is a well-defined multiplicative Dedekind-Hasse norm on D.