

1. Write each of the following complex numbers in the form $a + bi$.

(a): $-12e^{i\pi/4}$ (b): $e^{17i\pi} + 2e^{5i\pi/4}$

2. Let α and β be real numbers. You will show here that $e^{i\alpha}e^{i\beta} = e^{i(\alpha+\beta)}$ using Euler's formula ($e^{i\theta} = \cos(\theta) + i\sin(\theta)$).

First write $e^{i\alpha}$ and $e^{i\beta}$ using Euler's formula, then multiply the expressions and expand the result to put it into standard form $a + bi$. That is, multiply out $(\cos(\alpha) + i\sin(\alpha))(\cos(\beta) + i\sin(\beta))$, and put all real terms together and all imaginary terms together. Then show you get the same result when applying Euler's formula to $e^{i(\alpha+\beta)}$ by using the trigonometric angle sum formulas (which are $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ and $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$).

3. Find the general solution to each of the following homogeneous linear differential equations with constant coefficients. Use WolframAlpha or another mathematical tool to factor the characteristic polynomial.

(a): $y^{(5)} - 6y^{(4)} + 16y''' - 32y'' + 48y' - 32y = 0$

(b): $y^{(5)} - 11y^{(4)} + 66y''' - 230y'' + 481y' - 507y = 0$