These are optional problems for Math 215, which may be turned in on Wed., Apr. 26.

1. Let A be an m by k matrix with entries from $\mathbb{R}$. Let $\mathbf{r}$ be any vector in the row space of A, $R(A)$, and let $\mathbf{n}$ be any vector in the null space of A, $N(A)$. Show that $\mathbf{r} \cdot \mathbf{n} = 0$.

2. Let L be the following line in $\mathbb{R}^2$:

$$L = \text{span}\left\{ \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right\},$$

and let $P_L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which projects vectors in $\mathbb{R}^2$ onto the line L. Find the matrix (in terms of standard coordinates) which corresponds to P_L. That is, find a matrix B such that $P_L(\mathbf{v}) = B\mathbf{v}$ for any $\mathbf{v} \in \mathbb{R}^2$ given in standard coordinates.

3. Let P_L be as in problem 2 above. Show that

$$\mathbf{v}_1 = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \text{ and } \mathbf{v}_2 = \begin{pmatrix} -5/3 \\ 5 \end{pmatrix}$$

are linearly independent, while $P_L(\mathbf{v}_1)$ and $P_L(\mathbf{v}_2)$ are linearly dependent.

4. Let V and W be vector spaces over $\mathbb{R}$, and let $T : V \to W$ be a linear transformation which is *injective*. Show that if the vectors $\mathbf{v}_1, \ldots \mathbf{v}_k$ in V are linearly independent, then the vectors $T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)$ in W are also linearly independent.