Quiz 4, Math 211, Section 1 (Vinroot) Name: ______________________

(a): If V is a vector space, give the conditions which define a subset W to be a subspace of V.

Solution: W is a subspace of V if: (i) the zero vector $\mathbf{0}$ is in W, (ii) whenever $\mathbf{u}$ and $\mathbf{v}$ are in W, then $\mathbf{u}+\mathbf{v}$ is in W, and (iii) whenever $\mathbf{u}$ is in W and c is a scalar in $\mathbb{R}$, then $c\mathbf{u}$ is in W.

(b): Consider the following collection of vectors in $\mathbb{R}^2$:

$$H = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mid x_2 \geq 0 \right\},$$

so that H is the collection of all vectors in the upper-half of the plane, including the x_1-axis. Show that H satisfies two of the three conditions listed in **(a)**, but does not satisfy one of them, and conclude that H is not a subspace of $\mathbb{R}^2$.

Solution: First, $\mathbf{0}$ is in H since $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has second coordinate non-negative. Second, if $\mathbf{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix}$ are in H, so that $y \geq 0$ and $b \geq 0$, then

$$\mathbf{u}+\mathbf{v} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \end{bmatrix}$$

is also in H, since $y+b \geq 0$. So, the first two conditions listed in **(a)** hold. However, if we consider, say $\mathbf{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, which is in H since its second coordinate is $1 \geq 0$, and we take $c = -1$, then

$$c\mathbf{w} = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$

which is not an element of H. So, the third condition listed in **(a)** does not hold for H, and so H is not a subspace of $\mathbb{R}^2$.