Quiz 3 Solutions, Math 211, Section 1 (Vinroot)

(a): If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$, compute $2A - 3B$.

Solution: 

$$2A - 3B = 2 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - 3 \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 6 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 6 & -2 \end{bmatrix}.$$ 

(b): If $B$ is a $3 \times 7$ matrix, and $A$ is a $3 \times 5$ matrix, what is the size is $A^T B$? Explain briefly.

Solution: Since $A$ is $3 \times 5$, then $A^T$ is $5 \times 3$. Then $A^T B$ is a $5 \times 3$ matrix multiplied by a $3 \times 7$ matrix, which results in a $5 \times 7$ matrix. So $A^T B$ is a $5 \times 7$ matrix.

(c): If we were given $B$ and $A$ with sizes as in part (b), could we compute $BA$? Why or why not?

Solution: We could not compute $BA$. Since $B$ is a $3 \times 7$ matrix, and $A$ is a $3 \times 5$ matrix, then $BA$ is not defined.

(d): If $T : \mathbb{R}^2 \to \mathbb{R}^2$ and $S : \mathbb{R}^2 \to \mathbb{R}^2$ are linear transformations with standard matrices $C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, respectively, what is the standard matrix of the linear transformation $T \circ S : \mathbb{R}^2 \to \mathbb{R}^2$?

Solution: The standard matrix for $T \circ S$ is the product $CD$. In fact, this is how we motivated the definition of the product $CD$. We compute $CD$:

$$CD = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + (-1) \cdot 2 & 1 \cdot 0 + (-1) \cdot 1 \\ 0 \cdot 1 + 1 \cdot 2 & 0 \cdot 0 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}.$$