Quiz 2, Math 211, Section 1 (Vinroot) Name: ____________________

Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ -2 \\ 2 \end{bmatrix}.$$

(a): Using the information given and the fact that T is a linear transformation, find $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$, and explain.

Solution: Since T is a linear transformation, we know $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for any vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^2$. So, we have

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) + T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$
$$= \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -5 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}.$$

(b): Find the standard matrix for T, and briefly explain. Compute $T\left(\begin{bmatrix} -3 \\ 2 \end{bmatrix}\right)$ using the standard matrix.

Solution: We know that the standard matrix for T is the matrix $[T(\mathbf{e}_1) \;\; T(\mathbf{e}_2)]$. In part **(a)**, we computed that $T(\mathbf{e}_1) = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$, and part of our given information is that $T(\mathbf{e}_2) = \begin{bmatrix} -5 \\ -2 \\ 2 \end{bmatrix}$. Thus, the standard matrix for T is the following matrix, call it A:

$$A = \begin{bmatrix} -2 & -5 \\ 0 & -2 \\ 2 & 2 \end{bmatrix}.$$

Now, given any vector $\mathbf{x} \in \mathbb{R}^2$, we know $T(\mathbf{x}) = A\mathbf{x}$. To answer the last question, we compute:

$$T\left(\begin{bmatrix} -3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -2 & -5 \\ 0 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ -2 \end{bmatrix}.$$