

Quiz 6 **Solutions**, Math 112, Section 1 (Vinroot)

For each of the following, either find the value of the limit if the sequence converges, otherwise say why the sequence diverges. Show or explain all steps clearly for full credit.

1. $\lim_{n \rightarrow \infty} \frac{3n^3}{4n^3 - 2n^2}$

Solution: Using limit laws, we have

$$\lim_{n \rightarrow \infty} \frac{3n^3}{4n^3 - 2n^2} = \lim_{n \rightarrow \infty} \frac{3n^3 \cdot (1/n^3)}{(4n^3 - 2n^2) \cdot (1/n^3)} = \lim_{n \rightarrow \infty} \frac{3}{4 - 2/n} = \frac{3}{4 - \lim_{n \rightarrow \infty} (2/n)} = \frac{3}{4 - 0} = \frac{3}{4}.$$

2. $\lim_{n \rightarrow \infty} \ln(2 + (1/10)^n)$

Solution: We have $f(x) = \ln(x)$ is continuous for $x > 0$, and so

$$\lim_{n \rightarrow \infty} \ln(2 + (1/10)^n) = \ln\left(\lim_{n \rightarrow \infty} (2 + (1/10)^n)\right) = \ln\left(2 + \lim_{n \rightarrow \infty} ((1/10)^n)\right) = \ln(2),$$

since $0 < 1/10 < 1$, so $(1/10)^n$ converges to 0 as $n \rightarrow \infty$.

3. $\lim_{n \rightarrow \infty} (-1)^{n+1} \frac{8^n}{7^n}$

Solution: We note that $8^n/7^n = (8/7)^n$ diverges since $8/7 > 1$. The factor $(-1)^{n+1}$ alternates the signs of the terms, and so the sequence $\{(-1)^{n+1} \frac{8^n}{7^n}\}$ diverges, since the terms get larger and larger in absolute value, and also happen to alternate signs as they go. So this sequence **diverges**.