Quiz 4 **Solutions**, Math 112, Section 1 (Vinroot)

For each of the following find the value if it converges, or show that it diverges. Show all of your steps clearly to receive full credit.

1. $\displaystyle\int_0^2 x^{-4}\,dx$

Solution: The integrand $f(x) = x^{-4}$ is discontinuous at $x = 0$. The integral is then defined to be

$$\lim_{t\to 0^+}\left(\int_0^2 x^{-4}\,dx\right) = \lim_{t\to 0^+}\left(-\frac{1}{3}x^{-3}\Big]_t^2\right)$$
$$= \lim_{t\to 0^+}\left(-\frac{1}{24}+\frac{1}{3t^3}\right)$$

Since $\displaystyle\lim_{t\to 0^+}\frac{1}{3t^3} = \infty$, then this integral **diverges**.

2. $\displaystyle\int_1^\infty e^{-2x}\,dx$

Solution: Since we are integrating over an infinite interval, then by definition this improper integral is

$$\int_1^\infty e^{-2x}\,dx = \lim_{b\to\infty}\left(\int_1^b e^{-2x}\,dx\right)$$
$$= \lim_{b\to\infty}\left(-\frac{1}{2}e^{-2x}\Big]_1^b\right)$$
$$= \lim_{b\to\infty}\left(-\frac{1}{2}e^{-2b}+\frac{1}{2}e^{-2}\right)$$
$$= \lim_{b\to\infty}\left(-\frac{1}{2}e^{-2b}\right)+\frac{1}{2e^2}$$
$$= 0+\frac{1}{2e^2} = \frac{1}{2e^2}.$$

That is, the integral converges, and has value $\dfrac{1}{2e^2}$.