Quiz 3 **Solutions**, Math 112, Section 1 (Vinroot)

Compute each of the following. Show all of your steps clearly to receive full credit.

1. Find the value of $\displaystyle\int_0^1 \frac{2x^2 - x + 4}{(x-2)(x^2+1)}\,dx$.

Solution: We use partial fractions to first decompose the rational function we have to integrate:

$$\frac{2x^2 - x + 4}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1},$$

which, when getting a common denominator and setting numerators equal yields

$$2x^2 - x + 4 = A(x^2+1) + (Bx+C)(x-2).$$

We can let $x = 2$ in this equation, which gives $2(4) - 2 + 4 = A(2^2+1)$, so $10 = 5A$, and thus $A = 2$. To solve for B and C, we must multiply out the right hand side of the equation above and set coefficients of the two polynomials equal. We compute

$$2x^2 - x + 4 = Ax^2 + A + Bx^2 + Cx - 2Bx - 2C = (A+B)x^2 + (C-2B)x + A - 2C.$$

The coefficient of x^2 on each side must be equal, so $2 = A + B$, and $A = 2$, so $B = 0$. Setting the coefficient of x of each side equal gives $-1 = C - 2B = C - 2(0)$, and so $C = -1$. We now have

$$\begin{aligned}
\int_0^1 \frac{2x^2 - x + 4}{(x-2)(x^2+1)}\,dx &= \int_0^1 \left(\frac{2}{x-2} - \frac{1}{x^2+1}\right)\,dx \\
&= \left(2\ln|x-2| - \arctan(x)\right)\Big|_0^1 \\
&= \left(2\ln|1-2| - \arctan(1)\right) - \left(2\ln|0-2| - \arctan(0)\right) \\
&= 2\ln(1) - \frac{\pi}{4} - 2\ln(2) + 0 \\
&= -\frac{\pi}{4} - 2\ln(2).
\end{aligned}$$