Quiz 1 **Solutions**, Math 112, Section 1 (Vinroot)

Compute each of the following. Show all of your steps clearly to receive full credit.

**1.** Find the area of the region between the graphs of $y = e^x$ and $y = x$ from $x = 0$ to $x = 2$.

**Solution:** On the interval $[0, 2]$, we have $e^x > x$, meaning that the graph of $y = e^x$ is above the graph of $y = x$ from $x = 0$ to $x = 2$. Therefore the area is

$$\int_0^2 (e^x - x)\, dx = \left[e^x - \frac{1}{2}x^2\right]_0^2 = (e^2 - 2) - (e^0 - 0) = e^2 - 3.$$

**2.** Find the volume of the solid obtained by rotating the region from Problem **1** around the $x$-axis.

**Solution:** A cross-section of the solid perpendicular to the $x$-axis at a point $x_i$ will be a washer with outside radius $e^{x_i}$ and inside radius $x_i$. This washer of thickness $\Delta x$ will have volume equal to

$$\pi(e^{x_i})^2\Delta x - \pi(x_i)^2\Delta x = (\pi e^{2x_i} - \pi x_i^2)\Delta x.$$

When taking the limit of a Riemann sum of these volumes, we find the volume is

$$V = \int_0^2 (\pi e^{2x} - \pi x^2)\, dx = \int_0^2 \pi e^{2x}\, dx - \int_0^2 \pi x^2\, dx.$$

For the first integral, let $u = 2x$, so $du = 2\, dx$ and $\frac{1}{2}\, du = dx$, so $\int \pi e^{2x}\, dx = \int \frac{1}{2}\pi e^u\, du = \frac{\pi}{2}e^u + C = \frac{\pi}{2}e^{2x} + C$. Using this, we now have

$$V = \int_0^2 \pi e^{2x}\, dx - \int_0^2 \pi x^2\, dx = \left[\frac{\pi}{2}e^{2x}\right]_0^2 - \left[\frac{\pi}{3}x^3\right]_0^2 = \left(\frac{\pi}{2}e^4 - \frac{\pi}{2}\right) - \frac{8\pi}{3} = \frac{\pi e^4}{2} - \frac{19\pi}{6} = \frac{\pi}{6}(3e^4 - 19).$$