Quiz 5 **Solutions**, Math 112, Section 2 (Vinroot)

(a): Consider the differential equation $\frac{d^2y}{dx^2} - 4y = 0$. Show that for any constants c_1 and c_2, $y = c_1e^{2x} + c_2e^{-2x}$ is a solution.

Solution: If $y = c_1e^{2x} + c_2e^{-2x}$, then $\frac{dy}{dx} = 2c_1e^{2x} - 2c_2e^{-2x}$, and so

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(2c_1e^{2x} - 2c_2e^{-2x}) = 4c_1e^{2x} + 4c_2e^{-2x}.$$

Since $4y = 4c_1e^{2x} + 4c_2e^{-2x}$, then for $y = c_1e^{2x} + c_2e^{-2x}$, we have $\frac{d^2y}{dx^2} - 4y = 0$. So this is indeed a solution, for any constants c_1 and c_2.

(b): Consider the differential equation $\frac{dy}{dt} = 5(y-2)(y-1)$. Suppose $y(t)$ is a solution, and $y(t) < 1$ for all t. Explain why this must mean $y(t)$ is increasing at all t.

Solution: Since $y(t) < 1$ for all t, then $y(t) - 1 < 0$ for all t. Then also $y(t) - 2 < -1 < 0$ for all t. Then $(y-2)(y-1) > 0$. So we have

$$\frac{dy}{dt} = 5(y-2)(y-1) > 0,$$

which means that $y'(t) = \frac{dy}{dt} > 0$ for all t, so $y(t)$ is increasing at all t.