Quiz 2 Solutions, Math 112, Section 2 (Vinroot)

Show all steps of each of the following parts clearly.

(a): Compute \( \int \frac{\ln x}{x^2} \, dx \).

Solution: Use integration by parts, with \( u = \ln x \), so \( du = \frac{1}{x} \, dx \), and \( dv = \frac{1}{x^2} \, dx = x^{-2} \, dx \), so \( v = -x^{-1} \). Then,

\[
\int \frac{\ln x}{x^2} \, dx = (\ln x)(-x^{-1}) - \int -x^{-1} \frac{1}{x} \, dx = -\ln x + \int x^{-2} \, dx = -\frac{\ln x}{x} - \frac{1}{x} + C.
\]

(b): Compute the average value of the function \( f(x) = xe^x \) over the interval \([0, 1] \).

Solution: We know that the average value of the function is given by the integral

\[
\frac{1}{1-0} \int_0^1 xe^x \, dx = \int_0^1 xe^x \, dx.
\]

We compute this integral using integration by parts. If \( u = x \), then \( du = dx \), and if \( dv = e^x \, dx \), then \( v = e^x \). So, we have

\[
\int_0^1 xe^x \, dx = xe^x \bigg|_0^1 - \int_0^1 e^x \, dx
\]

\[
= (e^1 - 0e^0) - e^x \bigg|_0^1
\]

\[
= e - (e^1 - e^0) = 1.
\]

So, the average value of \( f(x) = xe^x \) over \([0, 1] \) is 1.