Quiz 2 **Solutions**, Math 112, Section 2 (Vinroot)

Show all steps of each of the following parts clearly.

(a): Compute $\int \frac{\ln x}{x^2}\, dx$.

Solution: Use integration by parts, with $u = \ln x$, so $du = \frac{1}{x}\, dx$, and $dv = \frac{1}{x^2}\, dx = x^{-2}\, dx$, so $v = -x^{-1}$. Then,

$$\int \frac{\ln x}{x^2}\, dx = (\ln x)(-x^{-1}) - \int -x^{-1}\frac{1}{x}\, dx = -\frac{\ln x}{x} + \int x^{-2}\, dx = -\frac{\ln x}{x} - \frac{1}{x} + C.$$

(b): Compute the average value of the function $f(x) = xe^x$ over the interval $[0, 1]$.

Solution: We know that the average value of the function is given by the integral

$$\frac{1}{1-0}\int_0^1 xe^x\, dx = \int_0^1 xe^x\, dx.$$

We compute this integral using integration by parts. If $u = x$, then $du = dx$, and if $dv = e^x\, dx$, then $v = e^x$. So, we have

$$\begin{aligned}\int_0^1 xe^x\, dx &= xe^x\Big|_0^1 - \int_0^1 e^x\, dx \\ &= (1e^1 - 0e^0) - e^x\Big|_0^1 \\ &= e - (e^1 - e^0) = 1.\end{aligned}$$

So, the average value of $f(x) = xe^x$ over $[0, 1]$ is 1.