Quiz 1 Solutions, Math 112, Section 2 (Vinroot)
Show all steps of each of the following parts clearly.

(a): Find the area of the region bounded by the graphs of \( f(x) = x^3 + 2 \), \( g(x) = x^2 + 1 \), \( x = 0 \), and \( x = 1 \).

Solution: Between \( x = 0 \) and \( x = 1 \), we have \( x^3 + 2 \geq x^2 + 1 \), and so the area of the region is given by (a sketch of the region here would be very useful):

\[
\int_0^1 ((x^3 + 2) - (x^2 + 1)) \, dx = \int_0^1 (x^3 - x^2 + 1) \, dx = \left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 + x \right]_0^1 = \frac{1}{4} - \frac{1}{3} + 1 = \frac{11}{12}.
\]

(b): Consider the region from (a), and the solid obtained by rotating it around the x-axis. Give the integral representing the volume of this solid, but do not evaluate.

Solution: Since we are rotating around the x-axis, we will use vertical rectangles, that is, integrate with respect to \( x \). Rotating the rectangle at the \( i \)th subinterval, with sample point \( x_i^* \), gives a washer with outer radius given by \( f(x_i^*) = (x_i^*)^3 + 2 \) and inner radius \( g(x_i^*) = (x_i^*)^2 + 1 \). The volume of this washer is then \( \pi f(x_i^*)^2 \Delta x - \pi g(x_i^*)^2 \Delta x \). Summing up these volumes and taking the limit gives that the volume is

\[
\lim_{n \to \infty} \sum_{i=1}^n \pi \left[ ((x_i^*)^3 + 2)^2 - ((x_i^*)^2 + 1)^2 \right] \Delta x = \int_0^1 \pi \left[ (x^3 + 2)^2 - (x^2 + 1)^2 \right] \, dx.
\]