Quiz 1 **Solutions**, Math 112, Section 2 (Vinroot)

Show all steps of each of the following parts clearly.

(a): Find the area of the region bounded by the graphs of $f(x) = x^3 + 2$, $g(x) = x^2 + 1$, $x = 0$, and $x = 1$.

Solution: Between $x = 0$ and $x = 1$, we have $x^3 + 2 \geq x^2 + 1$, and so the area of the region is given by (a sketch of the region here would be very useful):

$$\int_0^1 \left((x^3+2) - (x^2+1)\right) dx = \int_0^1 (x^3 - x^2 + 1)\, dx = \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 + x\right]_0^1 = \frac{1}{4} - \frac{1}{3} + 1 = \frac{11}{12}.$$

(b): Consider the region from **(a)**, and the solid obtained by rotating it around the x-axis. Give the integral representing the volume of this solid, but **do not evaluate**.

Solution: Since we are rotating around the x-axis, we will use vertical rectangles, that is, integrate with respect to x. Rotating the rectangle at the ith subinterval, with sample point x_i^*, gives a washer with outer radius given by $f(x_i^*) = (x_i^*)^3 + 2$ and inner radius $g(x_i^*) = (x_i^*)^2 + 1$. The volume of this washer is then $\pi f(x_i^*)^2 \Delta x - \pi g(x_i^*)^2 \Delta x$. Summing up these volumes and taking the limit gives that the volume is

$$\lim_{n \to \infty} \sum_{i=1}^{n} \pi \left[((x_i^*)^3 + 2)^2 - ((x_i^*)^2 + 1)^2)\right] \Delta x = \int_0^1 \pi \left[(x^3+2)^2 - (x^2+1)^2\right] dx.$$