Evaluate each of the following limits. Indicate any indeterminate form which appears at each step, and show all work clearly.

(a): \[ \lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x^2} \]

**Solution:** Since \( e^{2(0)} - 1 - 2(0) = 0 \) and \( 0^2 = 0 \), then this limit is indeterminate of form “0/0”. Applying L’Hospital’s rule yields

\[
\lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x^2} = \lim_{x \to 0} \frac{2e^{2x} - 2}{2x} = \lim_{x \to 0} \frac{e^{2x} - 1}{x}.
\]

Again, this limit is indeterminate of form “0/0” since \( e^{2(0)} - 1 = 0 \) and the denominator also goes to 0. Applying L’Hospital’s rule again gives

\[
\lim_{x \to 0} \frac{e^{2x} - 1}{x} = \lim_{x \to 0} \frac{2e^{2x}}{1} = 2e^{2(0)} = 2.
\]

So we have \( \lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x^2} = 2 \).

(b): \( \lim_{x \to 0^+} x^{5/2} \ln(x) \)

**Solution:** We know \( \lim_{x \to 0^+} \ln(x) = -\infty \), and since \( x^{5/2} \to 0 \) as \( x \to 0^+ \), then this limit is indeterminate of type “0 · ∞”. We transform the limit into a quotient by writing \( x^{5/2} = \frac{1}{x^{-5/2}} \). Then note that \( \lim_{x \to 0^+} \frac{\ln(x)}{x^{-5/2}} \) is an indeterminate form of type “∞/∞”, since \( x^{-5/2} \to \infty \) as \( x \to 0^+ \). We may thus apply L’Hospital’s rule to this limit. Doing this, we obtain

\[
\lim_{x \to 0^+} x^{5/2} \ln(x) = \lim_{x \to 0^+} \frac{\ln(x)}{x^{-5/2}} = \lim_{x \to 0^+} \frac{1/x}{-\frac{5}{2}x^{-7/2}} = \lim_{x \to 0^+} \left( -\frac{2}{5} x^{7/2} \right) = \lim_{x \to 0^+} \left( -\frac{2}{5} x^{5/2} \right) = 0.
\]

That is, \( \lim_{x \to 0^+} x^{5/2} \ln(x) = 0 \).