Quiz 8 **Solutions**, Math 111, Section 1 (Vinroot)

Evaluate each of the following limits. Indicate any indeterminate form which appears at each step, and show all work clearly.

(a): $\lim_{x\to 0} \dfrac{e^{2x} - 1 - 2x}{x^2}$

Solution: Since $e^{2(0)} - 1 - 2(0) = 0$ and $0^2 = 0$, then this limit is indeterminate of form "0/0". Applying L'Hospital's rule yields

$$\lim_{x\to 0} \frac{e^{2x} - 1 - 2x}{x^2} = \lim_{x\to 0} \frac{2e^{2x} - 2}{2x} = \lim_{x\to 0} \frac{e^{2x} - 1}{x}.$$

Again, this limit is indeterminate of form "0/0" since $e^{2(0)} - 1 = 0$ and the denominator also goes to 0. Applying L'Hospital's rule again gives

$$\lim_{x\to 0} \frac{e^{2x} - 1}{x} = \lim_{x\to 0} \frac{2e^{2x}}{1} = 2e^{2(0)} = 2.$$

So we have $\lim_{x\to 0} \dfrac{e^{2x} - 1 - 2x}{x^2} = 2$.

(b): $\lim_{x\to 0^+} x^{5/2} \ln(x)$

Solution: We know $\lim_{x\to 0^+} \ln(x) = -\infty$, and since $x^{5/2} \to 0$ as $x \to 0^+$, then this limit is indeterminate of type "$0 \cdot \infty$". We transform the limit into a quotient by writing $x^{5/2} = \frac{1}{x^{-5/2}}$. Then note that $\lim_{x\to 0^+} \dfrac{\ln(x)}{x^{-5/2}}$ is an indeterminate form of type "∞/∞", since $x^{-5/2} \to \infty$ as $x \to 0^+$. We may thus apply L'Hospital's rule to this limit. Doing this, we obtain

$$\lim_{x\to 0^+} x^{5/2} \ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{x^{-5/2}} = \lim_{x\to 0^+} \frac{1/x}{-\frac{5}{2} x^{-7/2}} = \lim_{x\to 0^+} \left(-\frac{2}{5} \frac{x^{7/2}}{x} \right) = \lim_{x\to 0^+} \left(-\frac{2}{5} x^{5/2} \right) = 0.$$

That is, $\lim_{x\to 0^+} x^{5/2} \ln(x) = 0$.