Quiz 7 **Solutions**, Math 111, Section 1 (Vinroot)

For both parts below, let $f(x) = 1 + 3x^2 - 2x^3$. Show all work clearly, with some explanation.
(a): Find all intervals where $f(x)$ is increasing, where it is decreasing, and find the locations of all local maxima and minima.

Solution: The function is increasing when $f'(x) > 0$, decreasing when $f'(x) < 0$, and there is a local extremum whenever $f'(x)$ changes sign, according to the first derivative test. We have $f'(x) = 6x - 6x^2 = 6x(1-x)$. The critical numbers are thus $x = 0$ and $x = 1$. We consider the signs of $f'(x)$ on the intervals defined by these critical numbers.

- When $x < 0$, $6x < 0$ and $1 - x > 0$, so $f'(x) = 6x(1-x) < 0$.
- When $0 < x < 1$, $6x > 0$ and $1 - x > 0$, so $f'(x) = 6x(1-x) > 0$.
- When $x > 1$, $6x > 0$ and $1 - x < 0$, so $f'(x) = 6x(1-x) < 0$.

So $f(x)$ is increasing when $0 < x < 1$ and decreasing when $x < 0$ or $x > 1$. By the first derivative test, there is a local minimum at $x = 0$ and a local maximum at $x = 1$. Since $f(0) = 1$ and $f(1) = 2$, then the local minimum is at $(0, 1)$ and the local maximum is at $(1, 2)$.

(b): Find all intervals where $f(x)$ is concave up, where it is concave down, and find the locations of all points of inflection.

Solution: The function is concave up when $f''(x) > 0$, concave down when $f''(x) < 0$, and points of inflection occur when $f''(x)$ changes sign. We have $f'(x) = 6x - 6x^2$, and so $f''(x) = 6 - 12x = 6(1-2x)$. The sign of $f''(x)$ depends only on the sign of $1-2x$, which is 0 when $x = 1/2$. When $x < 1/2$, $1 - 2x > 0$ and so $f''(x) > 0$. When $x > 1/2$, then $1 - 2x < 0$ and so $f''(x) < 0$. Thus the function is concave up when $x < 1/2$, concave down when $x > 1/2$, and there is a point of inflection at $x = 1/2$. Since $f(1/2) = 1 + (3/4) - (2/8) = 3/2$, then the point of inflection is at $(1/2, 3/2)$.