For both parts below, let \( f(x) = 1 + 3x^2 - 2x^3 \). Show all work clearly, with some explanation.

(a): Find all intervals where \( f(x) \) is increasing, where it is decreasing, and find the locations of all local maxima and minima.

**Solution:** The function is increasing when \( f'(x) > 0 \), decreasing when \( f'(x) < 0 \), and there is a local extremum whenever \( f'(x) \) changes sign, according to the first derivative test. We have \( f'(x) = 6x - 6x^2 = 6x(1 - x) \). The critical numbers are thus \( x = 0 \) and \( x = 1 \). We consider the signs of \( f'(x) \) on the intervals defined by these critical numbers.

- When \( x < 0 \), \( 6x < 0 \) and \( 1 - x > 0 \), so \( f'(x) = 6x(1 - x) < 0 \).
- When \( 0 < x < 1 \), \( 6x > 0 \) and \( 1 - x > 0 \), so \( f'(x) = 6x(1 - x) > 0 \).
- When \( x > 1 \), \( 6x > 0 \) and \( 1 - x < 0 \), so \( f'(x) = 6x(1 - x) < 0 \).

So \( f(x) \) is increasing when \( 0 < x < 1 \) and decreasing when \( x < 0 \) or \( x > 1 \). By the first derivative test, there is a local minimum at \( x = 0 \) and a local maximum at \( x = 1 \). Since \( f(0) = 1 \) and \( f(1) = 2 \), then the local minimum is at \((0, 1)\) and the local maximum is at \((1, 2)\).

(b): Find all intervals where \( f(x) \) is concave up, where it is concave down, and find the locations of all points of inflection.

**Solution:** The function is concave up when \( f''(x) > 0 \), concave down when \( f''(x) < 0 \), and points of inflection occur when \( f''(x) \) changes sign. We have \( f'(x) = 6x - 6x^2 \), and so \( f''(x) = 6 - 12x = 6(1 - 2x) \). The sign of \( f''(x) \) depends only on the sign of \( 1 - 2x \), which is 0 when \( x = 1/2 \). When \( x < 1/2 \), \( 1 - 2x > 0 \) and so \( f''(x) > 0 \). When \( x > 1/2 \), then \( 1 - 2x < 0 \) and so \( f''(x) < 0 \). Thus the function is concave up when \( x < 1/2 \), concave down when \( x > 1/2 \), and there is a point of inflection at \( x = 1/2 \). Since \( f(1/2) = 1 + (3/4) - (2/8) = 3/2 \), then the point of inflection is at \((1/2, 3/2)\).