Quiz 6 **Solutions**, Math 111, Section 1 (Vinroot)

(a): Find all critical numbers of the function \( g(x) = \sqrt[3]{x^2 - 2x} \).

**Solution:** We have \( g(x) = (x^2 - 2x)^{1/3} \), and so
\[
g'(x) = \frac{1}{3}(x^2 - 2x)^{-2/3}(2x - 2) = \frac{2(x - 1)}{3(x^2 - 2x)^{2/3}}.
\]
Critical numbers are those \( c \) such that \( g'(c) = 0 \) or \( g'(c) \) is undefined. Given \( g'(x) \) as computed above, we have \( g'(x) = 0 \) when the numerator is 0, and so \( c = 1 \) is a critical number. The derivative \( g'(x) \) is undefined when the denominator is 0, which happens exactly when \( x^2 - 2x = 0 \). This occurs when \( x(x - 2) = 0 \), which gives 0 and 2 as critical numbers. So the critical numbers are 0, 1, and 2.

(b): Find the absolute maximum and minimum values of the function \( g(x) \) from (a) on the interval \([0, 4]\).

**Solution:** Since the function is continuous for all values of \( x \), we may apply the closed interval method. So the maximum and minimum values occur at either a critical number, or an endpoint of the interval. The critical number \( c = 0 \) is also an endpoint, so this is being checked anyway. The values at the other two critical numbers are \( g(1) = \sqrt[3]{1 - 2} = \sqrt[3]{-1} = -1 \), and \( g(2) = \sqrt[3]{2^2 - 4} = 0 \). The values at the endpoints are \( g(0) = \sqrt[3]{0^2 - 0} = 0 \) and \( g(4) = \sqrt[3]{4^2 - 8} = \sqrt[3]{8} = 2 \). So the absolute maximum of the function on the interval \([0, 4]\) is \( g(4) = 2 \), and the absolute minimum is \( g(1) = -1 \).