Quiz 6 **Solutions**, Math 111, Section 1 (Vinroot)

(a): Find all critical numbers of the function $g(x) = \sqrt[3]{x^2 - 2x}$.

Solution: We have $g(x) = (x^2 - 2x)^{1/3}$, and so

$$g'(x) = \frac{1}{3}(x^2 - 2x)^{-2/3}(2x - 2) = \frac{2(x-1)}{3(x^2 - 2x)^{2/3}}.$$

Critical numbers are those c such that $g'(c) = 0$ or $g'(c)$ is undefined. Given $g'(x)$ as computed above, we have $g'(x) = 0$ when the numerator is 0, and so $c = 1$ is a critical number. The derivative $g'(x)$ is undefined when the denominator is 0, which happens exactly when $x^2 - 2x = 0$. This occurs when $x(x-2) = 0$, which gives 0 and 2 as critical numbers. So the critical numbers are 0, 1, and 2.

(b): Find the absolute maximum and minimum values of the function $g(x)$ from **(a)** on the interval $[0, 4]$.

Solution: Since the function is continuous for all values of x, we may apply the closed interval method. So the maximum and minimum values occur at either a critical number, or an endpoint of the interval. The critical number $c = 0$ is also an endpoint, so this is being checked anyway. The values at the other two critical numbers are $g(1) = \sqrt[3]{1-2} = \sqrt[3]{-1} = -1$, and $g(2) = \sqrt[3]{2^2 - 4} = 0$. The values at the endpoints are $g(0) = \sqrt[3]{0^2 - 0} = 0$ and $g(4) = \sqrt[3]{4^2 - 8} = \sqrt[3]{8} = 2$. So the absolute maximum of the function on the interval $[0, 4]$ is $g(4) = 2$, and the absolute minimum is $g(1) = -1$.