Quiz 5 **Solutions**, Math 111, Section 1 (Vinroot)

(a): Compute the derivative of the following function: $f(x) = \arctan(2x) + \log_5(\cos(x))$.

Solution: Recall that the derivative of $\arctan(x)$ is $1/(1+x^2)$ and the derivative of $\log_5(x)$ is $1/(x\ln(5))$. Using these together with the chain rule gives

$$\begin{aligned} f'(x) &= \frac{1}{1+(2x)^2}\cdot\frac{d}{dx}(2x) + \frac{1}{\cos(x)\ln(5)}\cdot\frac{d}{dx}(\cos(x)) \\ &= \frac{1}{1+4x^2}\cdot 2 + \frac{1}{\cos(x)\ln(5)}\cdot(-\sin(x)) \\ &= \frac{2}{1+4x^2} - \frac{\tan(x)}{\ln(5)}. \end{aligned}$$

(b): Compute the derivative of the following function: $y = x^{\tan(x)}$.

Solution: The main strategy is to first take the natural logarithm of both sides, and then differentiate. Taking the natural logarithm gives $\ln(y) = \ln(x^{\tan(x)}) = \tan(x)\ln(x)$. When we differentiate, the left side is $\frac{1}{y}\cdot y'$ by the chain rule, since the derivative of $\ln(x)$ is $1/x$. For the right side, we use the product rule, and we obtain

$$\frac{y'}{y} = \frac{d}{dx}(\tan(x))\cdot\ln(x) + \tan(x)\frac{d}{dx}(\ln(x)) = \sec^2(x)\ln(x) + \tan(x)\cdot\frac{1}{x}.$$

To obtain the derivative y', we multiply both sides by $y = x^{\tan(x)}$, and we obtain

$$y' = x^{\tan(x)}\left(\sec^2(x)\ln(x) + \frac{\tan(x)}{x}\right).$$