Quiz 4 **Solutions**, Math 111, Section 1 (Vinroot)

(a): Compute the derivative of the following function, making your steps clear:

$$f(x) = \tan(\sqrt{x}) + 5^{\sin(x)}$$

Solution: The function $f(x)$ is the sum of two functions, each of which is the composition of two functions, so we must use the Chain rule for each. Doing this, we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}(\tan(x^{1/2})) + \frac{d}{dx}(5^{\sin(x)}) \\ &= \sec^2(x^{1/2})\frac{d}{dx}(x^{1/2}) + 5^{\sin(x)}\ln(5)\frac{d}{dx}(\sin(x)) \\ &= \sec^2(x^{1/2})\frac{1}{2}x^{-1/2} + 5^{\sin(x)}\ln(5)\cos(x) \\ &= \frac{\sec^2(\sqrt{x})}{2\sqrt{x}} + 5^{\sin(x)}\ln(5)\cos(x). \end{aligned}$$

(b): Compute y' if $xy^2 - \cos(xy) = xe^y$, making your steps clear.

Solution: We use implicit differentiation, making sure we treat y as a function of x. Taking the derivative of both sides with respect to x, that is,

$$\frac{d}{dx}(xy^2) - \frac{d}{dx}(\cos(xy)) = \frac{d}{dx}(xe^y),$$

we use the product and chain rules to obtain

$$y^2 + x(2yy') + \sin(xy)(y + xy') = e^y + xe^y y'.$$

We multiply out the terms and move the summands with y' on one side (say the left side), and other terms on the right side:

$$2xyy' + xy'\sin(xy) - xy'e^y = e^y - y^2 - y\sin(xy).$$

Factor out y' from the left side to obtain

$$y'(2xy + x\sin(xy) - xe^y) = e^y - y^2 - y\sin(xy).$$

Solving for y' now gives our final answer as $y' = \dfrac{e^y - y^2 - y\sin(xy)}{2xy + x\sin(xy) - xe^y}$.