Quiz 3 **Solutions**, Math 111, Section 1 (Vinroot)

(a): Use the *limit definition* to compute the derivative of $f(x) = x^2 - x$. Make sure all of your steps are clear.

Solution: Using the definition of the derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h} \\ &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h} = \lim_{h \to 0} \frac{2xh + h^2 - h}{h} \\ &= \lim_{h \to 0} \frac{h(2x + h - 1)}{h} = \lim_{h \to 0} (2x + h - 1) = 2x - 1. \end{aligned}$$

So, the function $f(x) = x^2 - x$ has derivative $f'(x) = 2x - 1$.

(b): Use your answer to part **(a)** to find the equation of the tangent line to $f(x) = x^2 - x$ at the point $(3, 6)$.

Solution: From the meaning of the derivative, the slope of the tangent line at $(3, 6)$ is $f'(3)$. Using $f'(x) = 2x - 1$ from part **(a)**, we have $f'(3) = 2(3) - 1 = 5$. Since a point on this tangent line is $(3, 6)$, where $f(3) = 6$, the equation of the tangent line is given by

$$5 = \frac{y - 6}{x - 3}, \quad \text{which gives} \quad 5x - 15 = y - 6, \quad \text{or} \quad y = 5x - 9.$$

That is, the equation of the tangent line is $y = 5x - 9$.