Quiz 1 **Solutions**, Math 111, Section 1 (Vinroot)

(a): Compute the following limit if it exists, making your steps clear: $\lim_{x\to -2} \frac{\sqrt{x+6}-2}{2x+4}$.

Solution: We note that we cannot directly substitute $x = -2$ since the numerator and denominator would be 0. To get a cancellation, we rationalize the numerator by multiplying the numerator and denominator by $\sqrt{x+6}+2$. This gives:

$$\lim_{x\to -2} \frac{\sqrt{x+6}-2}{2x+4} \cdot \frac{\sqrt{x+6}+2}{\sqrt{x+6}+2} = \lim_{x\to -2} \frac{(\sqrt{x+6})^2 - 2^2}{(2x+4)(\sqrt{x+6}+2)} = \lim_{x\to -2} \frac{(x+6)-4}{(2x+4)(\sqrt{x+6}+2)}.$$

When we simplify the numerator, and factor the term $2x+4$ in the denominator, we are able to cancel the factor which was preventing direct substitution:

$$\lim_{x\to -2} \frac{(x+6)-4}{(2x+4)(\sqrt{x+6}+2)} = \lim_{x\to -2} \frac{x+2}{2(x+2)(\sqrt{x+6}+2)} = \lim_{x\to -2} \frac{1}{2(\sqrt{x+6}+2)}.$$

We may finally directly substitute by the properties of limits (noting that the term under the square root goes to a positive number, and the denominator now goes to a nonzero number), which gives:

$$\lim_{x\to -2} \frac{\sqrt{x+6}-2}{2x+4} = \lim_{x\to -2} \frac{1}{2(\sqrt{x+6}+2)} = \frac{1}{2(\sqrt{-2+6}+2)} = \frac{1}{2(4)} = \frac{1}{8}.$$

(b): Define the function $f(x)$ as follows:

$$f(x) = \begin{cases} x^3 - 4 & \text{if } x < 1 \\ x^2 - cx + 1 & \text{if } x > 1. \end{cases}$$

Determine the value of c so that $\lim_{x\to 1} f(x)$ exists, by calculating one-sided limits, and briefly explain.

Solution: In order for $\lim_{x\to 1} f(x)$, we need $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$. Computing each one-sided limit, using the definition of the function and direction substitution (since we have polynomials), gives

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^3 - 4) = 1^3 - 4 = -3, \text{ and}$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^2 - cx + 1) = 1^2 - c + 1 = 2 - c.$$

So, in order for $\lim_{x\to 1} f(x)$ to exist, we need $2 - c = -3$, which gives $c = 5$.