Quiz 7 **Solutions**, Math 111, Section 4 (Vinroot)

For both parts of this problem, let $f(x) = x^3 - 12x + 2$. Make sure your work is clear.

(a): Find the intervals on which $f(x)$ is increasing, and where it is decreasing, and give the locations of the local maxima and minima.

Solution: The function is increasing whenever $f'(x) > 0$ and decreasing whenever $f'(x) < 0$. We have $f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x-2)(x+2)$. We consider possible sign changes of $f'(x)$ when $x = -2$ or $x = 2$ since these are the critical numbers.

- When $x < -2$, we have $x - 2 < 0$ and $x + 2 < 0$, and so $f'(x) = 3(x-2)(x+2) > 0$.
- When $-2 < x < 2$, we have $x - 2 < 0$ and $x + 2 > 0$, and so $f'(x) = 3(x-2)(x+2) < 0$.
- When $x > 2$, we have $x - 2 > 0$ and $x + 2 > 0$, and so $f'(x) = 3(x-2)(x+2) > 0$.

So, $f(x)$ is increasing if $x < -2$ or $x > 2$, and $f(x)$ is increasing if $-2 < x < 2$. Since $f(x)$ goes from increasing to decreasing at $x = -2$, then there is a local maximam and $x = -2$. Since $f(x)$ goes from decreasing to increasing at $x = 2$, then there is a local minimum at $x = 2$.

(b): Find the intervals on which $f(x)$ is concave up, and where $f(x)$ is concave down, and give the locations of points of inflection.

Solution: The function is concave up whenever $f''(x) > 0$ and concave down whenever $f''(x) < 0$, and there are points of inflection whenever the concavity changes. We have $f''(x) = 6x$, and so $f''(x) < 0$ if $x < 0$ and $f''(x) > 0$ if $x > 0$. Thus $f(x)$ is concave up when $x > 0$, concave down when $x < 0$, and there is a point of inflection at $x = 0$.