Quiz 6 **Solutions**, Math 111, Section 4 (Vinroot)

(a): Find all critical numbers of $f(x) = xe^{-2x}$, with a brief explanation.

Solution: The critical numbers for a function $f(x)$ are numbers in the domain of $f(x)$ such that $f'(x)$ is either 0 or undefined. From the product rule and chain rule, we have

$$f'(x) = e^{-2x} - 2xe^{-2x}.$$

Factoring out e^{-2x}, we have $f'(x) = e^{-2x}(1-2x)$. Note that $f'(x)$ is defined for all x, so the critical numbers are those values for which $f'(x) = 0$. Since e^{-2x} is never 0, then $f'(x)$ can only equal 0 if $1 - 2x = 0$. That is, the only critical number of $f(x)$ is $x = 1/2$.

(b): Using your work from **(a)**, find the absolute maximum and minimum values of $f(x) = xe^{-2x}$ on the interval $[0, 1]$, with a brief explanation.

Solution: Our function is continuous for all input, and so it is continuous on the interval $[0, 1]$. So we may use the closed interval method to find the maximum and minimum values of $f(x)$ on $[0, 1]$, and they must occur at either a critical number or an endpoint. The values at these numbers, $x = 1/2, 0$, and 1 are:

$$f(1/2) = \frac{1}{2}e^{-1} = \frac{1}{2e}, \quad f(0) = 0, \quad f(1) = e^{-2} = \frac{1}{e^2}.$$

The minimum of these values is 0, so the absolute minimum occurs at $(0, 0)$. Of the other two values, we know $e > 2$, so $e^2 > 2e$, and so $\frac{1}{e^2} < \frac{1}{2e}$. So the absolute maximum occurs at $(\frac{1}{2}, \frac{1}{2e})$.