(a): If \( f(x) = \sin(x)\sin(x) \) compute \( f'(x) \), showing steps clearly.

**Solution:** Since this function is of the form \( g(x)^h(x) \), we take the natural logarithm of both sides first, which gives \( \ln(f(x)) = \ln(\sin(x)^{\sin(x)}) = \sin(x)\ln(\sin(x)) \). Now we take the derivative of both sides, noting that \( \frac{d}{dx}(\ln(f(x))) = \frac{f'(x)}{f(x)} \) from the chain rule. This gives

\[
\frac{f'(x)}{f(x)} = \frac{d}{dx}(\sin(x)\ln(\sin(x)))
\]

\[
= \cos(x)\ln(\sin(x)) + \sin(x) \cdot \frac{1}{\sin(x)} \cdot \cos(x) = \cos(x)\ln(\sin(x)) + \cos(x).
\]

We obtain the final answer by multiplying both sides by \( f(x) \), so

\[
f'(x) = f(x) (\cos(x)\ln(\sin(x)) + \cos(x)) = \sin(x)^{\sin(x)} (\cos(x)\ln(\sin(x)) + \cos(x))
\]

(b): A colony of bacteria cells is growing exponentially, and starts as a population of 1.1 million cells. After 30 minutes, there are 2.3 millions cells. Find a formula for the number of bacteria cells there are (in millions of cells) after \( t \) minutes. Leave your answer in terms of logarithms and/or exponentials, and show all steps of your solution clearly.

**Solution:** We let \( P(t) \) be the population of the colony in millions of cells at time \( t \) in minutes. We take \( t = 0 \) to be the initial time, so that \( P(0) = 1.1 \). Since we are assuming exponential growth, we know that \( P(t) = P(0)e^{kt} = 1.1e^{kt} \) for some \( k \). We calculate \( k \) by using the fact that \( P(30) = 2.3 \). We have

\[
P(30) = 2.3 = 1.1e^{30k} \implies \frac{2.3}{1.1} = e^{30k} \implies \ln\left(\frac{2.3}{1.1}\right) = 30k \implies k = \frac{1}{30}\ln\left(\frac{2.3}{1.1}\right).
\]

We substitute this into our equation for \( P(t) \) and obtain the final equation

\[
P(t) = 1.1e^{\frac{t}{30}\ln\left(\frac{2.3}{1.1}\right)}.
\]