Quiz 4 **Solutions**, Math 111, Section 4 (Vinroot)

(a): Compute y', showing steps carefully, if

$$x^2\sin(y^2)+e^y=2x^2y.$$

Solution: We use implicit differentiation, and take the derivative with respect to x on each side. Recall that by the chain rule, since y is a function of x, then factors of y' appear whenever we take derivatives of expressions with y in them. For the left side, we have $\frac{d}{dx}(2x^2y)=4xy+2x^2y'$, and for the right side we have to use the product rule and the chain rule for the first summand:

$$\frac{d}{dx}\left(x^2\sin(y^2)+e^y\right)=2x\sin(y^2)+x^2\cos(y^2)2yy'+e^yy'.$$

Equating these, and then moving all terms with a y' on one side yields:

$$2x\sin(y^2)+2yy'x^2\cos(y^2)+y'e^y=4xy+2x^2y' \quad\Longrightarrow\quad 2yy'x^2\cos(y^2)+y'e^y-2x^2y'=4xy-2x\sin(y^2).$$

Now factor out a y' from the left side, and divide by the rest to solve for y', and we finally obtain

$$y'=\frac{4xy-2x\sin(y^2)}{2yx^2\cos(y^2)+e^y-2x^2}.$$

(b): Find the derivative of the following function, showing steps clearly:

$$f(x)=\ln(\sin^{-1}(x))+\tan^{-1}(e^{2x})$$

Solution: We apply the chain rule, along with the derivatives $\frac{d}{dx}(\ln(x))=1/x$, $\frac{d}{dx}(\sin^{-1}(x))=1/\sqrt{1-x^2}$, and $\frac{d}{dx}(\tan^{-1}(x))=1/(1+x^2)$. We compute that

$$\begin{aligned}f'(x)&=\frac{1}{\sin^{-1}(x)}\frac{d}{dx}(\sin^{-1}(x))+\frac{1}{1+(e^{2x})^2}\frac{d}{dx}(e^{2x})\\&=\frac{1}{\sin^{-1}(x)}\frac{1}{\sqrt{1-x^2}}+\frac{1}{1+e^{4x}}\cdot 2e^{2x}\\&=\frac{1}{(\sqrt{1-x^2})\sin^{-1}(x)}+\frac{2e^{2x}}{1+e^{4x}}.\end{aligned}$$