Quiz 2 **Solutions**, Math 111, Section 4 (Vinroot)

(a): Compute the following limit, showing steps clearly:

$$\lim_{x\to-\infty} \frac{3x^2 - 2x^{4/3} - 5}{2x^2 + 3x^{2/5} - x + 2}$$

Solution: The highest power of x in the denominator is x^2, and so we multiply the numerator and denominator of the expression by $1/x^2$. Note that we have to pay attention to the fractional exponents when we simplify. Doing this and applying properties of limits, we obtain

$$\lim_{x\to-\infty} \frac{3x^2 - 2x^{4/3} - 5}{2x^2 + 3x^{2/5} - x + 2} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to-\infty} \frac{3 - \frac{2}{x^{2/3}} - \frac{5}{x^2}}{2 + \frac{3}{x^{8/5}} - \frac{1}{x} + \frac{2}{x^2}}$$

$$= \frac{3 - 2\lim_{x\to-\infty}\frac{1}{x^{2/3}} - 5\lim_{x\to-\infty}\frac{1}{x^2}}{2 + 3\lim_{x\to-\infty}\frac{1}{x^{8/5}} - \lim_{x\to-\infty}\frac{1}{x} + 2\lim_{x\to-\infty}\frac{1}{x^2}}.$$

We know that $\lim_{x\to-\infty}\frac{1}{x^2} = \lim_{x\to-\infty}\frac{1}{x} = 0$, and since $x^{8/5}$ and $x^{2/3}$ are both defined when $x < 0$ (since fifth roots and cube roots are) then also we have $\lim_{x\to-\infty}\frac{1}{x^{8/5}} = \lim_{x\to-\infty}\frac{1}{x^{2/3}} = 0$. So we have

$$\lim_{x\to-\infty} \frac{3x^2 - 2x^{4/3} - 5}{2x^2 + 3x^{2/5} - x + 2} = \frac{3}{2}.$$

(b): Compute the following limit, showing steps clearly:

$$\lim_{x\to\infty}\left(\sqrt{9x^2+5x} - 3x\right)$$

Solution: We rationalize the expression by multiplying the numerator and denominator by $\sqrt{9x^2+5x} + 3x$. Doing this, we get

$$\lim_{x\to\infty}\left(\sqrt{9x^2+5x} - 3x\right) \cdot \frac{\sqrt{9x^2+5x}+3x}{\sqrt{9x^2+5x}+3x} = \lim_{x\to\infty}\frac{9x^2+5x-9x^2}{\sqrt{9x^2+5x}+3x} = \lim_{x\to\infty}\frac{5x}{\sqrt{9x^2+5x}+3x}.$$

Now, the "effective degree" of the denominator is 1, since there is an x^2 term under the square root (which acts as the exponent $1/2$). So, we multiply the numerator and denominator by $1/x$. We have to bring the $1/x$ into the square root, and since $x \to \infty$, then $x > 0$, and so $1/x = \sqrt{1/x^2}$. Then apply properties of limits. Doing all of this, we have

$$\lim_{x\to\infty}\frac{5x}{\sqrt{9x^2+5x}+3x} \cdot \frac{1/x}{1/x} = \lim_{x\to\infty}\frac{\frac{5x}{x}}{\sqrt{\frac{9x^2}{x^2}+\frac{5x}{x^2}}+\frac{3x}{x}} = \lim_{x\to\infty}\frac{5}{\sqrt{9+\frac{5}{x}}+3}$$

$$= \frac{5}{\sqrt{9+\lim_{x\to\infty}\frac{5}{x}}+3} = \frac{5}{\sqrt{9+0}+3} = \frac{5}{6}.$$

So the limit exists and it has value $\frac{5}{6}$.