(a): Compute \( f'(x) \) if \( f(x) = \sin(3^x) + \ln(\sqrt{x} + x^2) \).

Solution: We use the chain rule for each of the two functions which sum to \( f(x) \). We rewrite \( \sqrt{x} = x^{1/2} \), and we recall that \( \frac{d}{dx}(3^x) = 3^x \ln(3) \) and \( \frac{d}{dx}(\ln(x)) = 1/x \). We have

\[
f'(x) = \cos(3^x) \frac{d}{dx}(3^x) + \frac{1}{x^{1/3} + x^2} \frac{d}{dx}(x^{1/3} + x^2)
\]
\[
= \cos(3^x)(3^x \ln(3)) + \frac{1}{x^{1/3} + x^2} \left( \frac{1}{3}x^{-2/3} + 2x \right).
\]

That is, we have

\[
f'(x) = 3^x \ln(3) \cos(3^x) + \frac{1}{3}x^{-2/3} + 2x \frac{1}{x^{1/3} + x^2}.
\]

(b): Compute \( y' \), given that \( \cos(x^2y) = \sin(xy^2) - ye^x \).

Solution: We use implicit differentiation. That is, we take the derivative with respect to \( x \) on both sides, and keep in mind that \( y \) is implicitly a function of \( x \) so the chain rule must be applied in each case there is a \( y \) in an expression. For example, we have \( \frac{d}{dx}(y^2) = 2yy' \). Differentiating, we have:

\[
\frac{d}{dx}(\cos(x^2y)) = \frac{d}{dx}(\sin(xy^2) - ye^x)
\]
\[
-\sin(x^2y)(2xy + x^2y') = \cos(xy^2)(y^2 + 2yy'x) - y'e^x - ye^x
\]

Now that the derivative is taken, we multiply out each side, and move all summands with a \( y' \) term to one side, and all other terms to the other side:

\[
-2xy \sin(x^2y) - y'x^2 \sin(x^2y) = y^2 \cos(xy^2) + 2xyy' \cos(xy^2) - y'e^x - ye^x
\]
\[
y'e^x - y'x^2 \sin(x^2y) - 2xyy' \cos(xy^2) = 2xy \sin(x^2y) + y^2 \cos(xy^2) - ye^x
\]
\[
y'(e^x - x^2 \sin(x^2y) - 2xy \cos(xy^2)) = 2xy \sin(x^2y) + y^2 \cos(xy^2) - ye^x
\]

Solving for \( y' \) gives

\[
y' = \frac{2xy \sin(x^2y) + y^2 \cos(xy^2) - ye^x}{e^x - x^2 \sin(x^2y) - 2xy \cos(xy^2)}.
\]