Quiz 3 **Solutions**, Math 111, Section 5 (Vinroot)

(a): Use the *limit definition of derivative* to calculate the derivative of $f(x) = \dfrac{-2}{x-3}$.

Solution: By definition of derivative, we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{-2}{x+h-3} - \frac{-2}{x-3}}{h}.$$

We now get a common denominator, and simplify the expression in the limit until we can cancel a factor of h:

$$f'(x) = \lim_{h \to 0} \frac{\frac{-2(x-3)}{(x-3)(x+h-3)} - \frac{-2(x+h-3)}{(x-3)(x+h-3)}}{h} = \lim_{h \to 0} \frac{-2(x-3) + 2(x+h-3)}{h(x-3)(x+h-3)} = \lim_{h \to 0} \frac{2h}{h(x-3)(x+h-3)}.$$

Now we cancel h, and use continuity to find the limit as $h \to 0$:

$$f'(x) = \lim_{h \to 0} \frac{2}{(x-3)(x+h-3)} = \frac{2}{(x-3)^2}.$$

So $f'(x) = \dfrac{2}{(x-3)^2}$.

(b): Find the equation of the line tangent to the curve $g(x) = 2x^2 - \sqrt{x} + 1$ at $x = 1$ (use formulas for derivatives here, not the limit definition).

Solution: We have $g(x) = 2x^2 - x^{1/2} + 1$. Using our formulas for derivatives that we have developed, we have:

$$g'(x) = 2\frac{d}{dx}(x^2) - \frac{d}{dx}(x^{1/2}) + \frac{d}{dx}(1) = 4x - \frac{1}{2}x^{-1/2}.$$

So, the slope of the tangent line at $x = 1$ is $g'(1) = 4(1) - \frac{1}{2}(1)^{-1/2} = 4 - \frac{1}{2} = \frac{7}{2}$. A point through which the line passes is $(1, g(1))$, and $g(1) = 2(1^2) - 1^{1/2} + 1 = 2$. So the equation of the tangent line at $x = 1$ is given by:

$$\frac{7}{2} = \frac{y-2}{x-1}, \quad \text{or} \quad y = \frac{7}{2}x - \frac{3}{2}.$$