Practice Quiz 8 **Solutions**, Math 111, Section 4 (Vinroot)

1. Find a function $f(x)$ which satisfies $f'(x) = \dfrac{2}{\sqrt{1-x^2}} + x + 3^x$ and $f(0) = 0$.

Solution: By remembering the antiderivative formulas we obtained from derivatives, we find the general antiderivative to be

$$f(x) = 2\arcsin(x) + \frac{1}{2}x^2 + \frac{3^x}{\ln(3)} + C,$$

where C is some constant. We use the condition $f(0) = 0$ to solve for C. By plugging in $x = 0$, we have

$$f(0) = 2\arcsin(0) + \frac{1}{2}0^2 + \frac{3^0}{\ln(3)} + C = 2(0) + 0 + \frac{1}{\ln(3)} + C,$$

since $\arcsin(0) = 0$. Since $f(0) = 0$, we have $\frac{1}{\ln(3)} + C = 0$, so $C = -1/\ln(3)$. Our final answer is then

$$f(x) = 2\arcsin(x) + \frac{1}{2}x^2 + \frac{3^x}{\ln(3)} - \frac{1}{\ln(3)}.$$

2. Suppose a particle moves along a line with acceleration given by $a(t) = 6t - 8$ at time t, with positions $s(0) = 1$ and $s(1) = -2$. Find the equation of the position function $s(t)$.

Solution: We take one antiderivative to find the velocity function $v(t)$, and one more antiderivative to find the position function $s(t)$. We use the given values of $s(t)$ to solve for constants. Taking one antiderivative, we have $v(t) = 3t^2 - 8t + C$ for some constant C. Since we do not have any velocity values, we cannot solve for C yet. Taking one more antiderivative gives $s(t) = t^3 - 4t^2 + Ct + D$ for some other constant D. Since $s(0) = 1$, we plug in $t = 0$ to find $s(0) = 0^3 - 4(0^2) + C(0) + D = D$, and since $s(0) = 1$ we have $D = 1$. So $s(t) = t^3 - 4t^2 + Ct + 1$. We finally use $s(1) = -2$ to solve for C, where $s(1) = 1^3 - 4(1^2) + C(1) + 1 = 1 - 4 + C + 1 = -2 + C$. Since $s(1) = -2$, we have $-2 + C = -2$, and so $C = 0$. Our final answer is now

$$s(t) = t^3 - 4t^2 + 1.$$