Quiz 8, Math 111 **Solutions**, Section 5 (Vinroot)

For each of the following, say what indeterminate form appears (at any step when there is one), and evaluate.

1. $\displaystyle\lim_{x\to 0} \frac{xe^x}{1-e^x}$

Solution: First, $xe^x \to 0$ and $1 - e^x \to 0$ as $x \to 0$, and so this is an indeterminate form of type $\frac{0}{0}$. So, we can apply L'Hospital's rule, and we get the following:

$$\lim_{x\to 0} \frac{xe^x}{1-e^x} = \lim_{x\to 0} \frac{xe^x + e^x}{-e^x} = \lim_{x\to 0} \frac{e^x(x+1)}{-e^x} = \lim_{x\to 0} -(x+1) = -1.$$

Note that after applying L'Hospital's rule, there is a common factor of e^x in the numerator and denominator that can be cancelled.

2. $\displaystyle\lim_{x\to 0^+} \left(\frac{e^x}{x} - \left(\frac{1}{x} + x^{1/2}\right)\right)$

Solution: As $x \to 0^+$, since $e^0 = 1$, we have $\frac{e^x}{x} \to \infty$. Since $x^{1/2} \to 0$ as $x \to 0^+$, then $(\frac{1}{x} + x^{1/2}) \to \infty$. So, our limit is the indeterminate form of type $\infty - \infty$. If we want to apply L'Hospital's rule, we have to turn this into an indeterminate quotient. Getting a common denominator, we have

$$\frac{e^x}{x} - \left(\frac{1}{x} + x^{1/2}\right) = \frac{e^x}{x} - \frac{1}{x} - \frac{x^{3/2}}{x} = \frac{e^x - 1 - x^{3/2}}{x}.$$

So,

$$\lim_{x\to 0^+} \left(\frac{e^x}{x} - \left(\frac{1}{x} + x^{1/2}\right)\right) = \lim_{x\to 0^+} \frac{e^x - 1 - x^{3/2}}{x},$$

which is an indeterminate form of type $\frac{0}{0}$. Applying L'Hospital's rule yields

$$\lim_{x\to 0^+} \frac{e^x - 1 - x^{3/2}}{x} = \lim_{x\to 0^+} \frac{e^x - \frac{3}{2}x^{1/2}}{1} = e^0 - \frac{3}{2}0^{1/2} = 1.$$