Quiz 7 **Solutions**, Math 111, Section 5 (Vinroot) Name: ______________________

1. Find all x for which the function $f(x) = x^3 - 2x^2 + 6x - 3$ is concave down.

Solution: We want all x such that $f''(x) < 0$. We have $f'(x) = 3x^2 - 4x + 6$, and $f''(x) = 6x - 4$. Then $f''(x) < 0$ when $6x - 4 < 0$, which happens when $x < 2/3$. So f is concave down when $x < 2/3$.

2. Find the maximum and minimum values attained by the function $g(x) = \dfrac{x^{1/3}}{x+2}$ on the interval $[-1, 8]$.

Solution: We must find all critical numbers of g, and then check the values of $g(x)$ at the critical numbers and the endpoints of the interval $[-1, 8]$. First, we compute $g'(x)$:

$$g'(x) = \frac{\frac{1}{3}x^{-2/3}(x+2) - x^{1/3}}{(x+2)^2}.$$

We may simplify a bit by multiplying the numerator and denominator of $g'(x)$ by $3x^{2/3}$ to clear the denominators on the top:

$$g'(x) = \frac{x + 2 - 3x}{3x^{2/3}(x+2)^2} = \frac{2 - 2x}{3x^{2/3}(x+2)^2}.$$

So $g'(x)$ is undefined if $x = 0$ or $x = -2$, and $g'(x) = 0$ if $x = 1$. So the critical numbers in the interval $[-1, 8]$ are 0 and 1. Now we compute values of $g(x)$ at the critical numbers and endpoints.

$$g(-1) = \frac{-1}{1} = -1, \quad g(0) = 0, \quad g(1) = \frac{1}{3}, \quad g(8) = \frac{8^{1/3}}{8+2} = \frac{2}{10} = \frac{1}{5}.$$

Thus, the minimum occurs at $x = -1$, where $g(-1) = -1$, and the maximum occurs at $x = 1$, where $g(1) = 1/3$.