Show all steps in the following, but do not simplify.

1. Compute the derivative of the following function: \( f(x) = \sin(e^x + x^2) + \sqrt{e^{\cos(x)}}. \)

**Solution:** We must use the Chain rule for each of the summands in the sum. In the first term, \( \sin(x) \) is the “outside” function, with \( e^x + x^2 \) being plugged into it. In the second term, there are three functions, with \( \cos(x) \) being plugged into \( e^x \), and their composition plugged into \( \sqrt{x} = x^{1/2} \). The derivative is then given by

\[
f'(x) = (\cos(e^x + x^2))(e^x + 2x) + \frac{1}{2}(e^{\cos(x)})^{-1/2}e^{\cos(x)}(-\sin(x)).
\]

While you do not need to simplify from here, it might be a good exercise to see if you can rewrite the expression as follows:

\[
f'(x) = e^x \cos(e^x + x^2) + 2x \cos(e^x + x^2) - \frac{\sin(x)}{2} \sqrt{e^{\cos(x)}}.
\]

2. Find \( y' \) if \( x^3y + \sin(y^2) = 2 - e^x + 3y^2 \).

**Solution:** We take the derivative with respect to \( x \) on both sides:

\[
\frac{d}{dx}(x^3y + \sin(y^2)) = \frac{d}{dx}(2 - e^x + 3y^2).
\]

Keeping in mind that \( y \) is implicitly a function of \( x \), we apply the Chain rule, along with the product rule for the \( x^3y \) term, to obtain

\[
3x^2y + x^3y' + \cos(y^2)2yy' = -e^x + 6yy'.
\]

We now move all summands including a \( y' \) term on one side to solve for \( y' \):

\[
3x^2y + e^x = 6yy' - x^3y' - \cos(y^2)2yy'
\]

\[
3x^2y + e^x = y'(6y - x^3 - 2y \cos(y^2)).
\]

We can finally obtain the following expression for \( y' \):

\[
y' = \frac{3x^2y + e^x}{6y - x^3 - 2y \cos(y^2)}.
\]