Quiz 4 **Solutions**, Math 111, Section 5 (Vinroot)

Show all steps in the following, but do not simplify.

1. Compute the derivative of the following function: $f(x) = \sin(e^x + x^2) + \sqrt{e^{\cos(x)}}$.

Solution: We must use the Chain rule for each of the summands in the sum. In the first term, $\sin(x)$ is the "outside" function, with $e^x + x^2$ being plugged into it. In the second term, there are three functions, with $\cos(x)$ being plugged into e^x, and their composition plugged into $\sqrt{x} = x^{1/2}$. The derivative is then given by

$$f'(x) = (\cos(e^x + x^2))(e^x + 2x) + \frac{1}{2}(e^{\cos(x)})^{-1/2}e^{\cos(x)}(-\sin(x)).$$

While you do not need to simplify from here, it might be a good exercise to see if you can rewrite the expression as follows:

$$f'(x) = e^x \cos(e^x + x^2) + 2x\cos(e^x + x^2) - \frac{\sin(x)}{2}\sqrt{e^{\cos(x)}}.$$

2. Find y' if $x^3y + \sin(y^2) = 2 - e^x + 3y^2$.

Solution: We take the derivative with respect to x on both sides:

$$\frac{d}{dx}(x^3y + \sin(y^2)) = \frac{d}{dx}(2 - e^x + 3y^2).$$

Keeping in mind that y is implicitly a function of x, we apply the Chain rule, along with the product rule for the x^3y term, to obtain

$$3x^2y + x^3y' + \cos(y^2)2yy' = -e^x + 6yy'.$$

We now move all summands including a y' term on one side to solve for y':

$$3x^2y + e^x = 6yy' - x^3y' - \cos(y^2)2yy'$$

$$3x^2y + e^x = y'(6y - x^3 - 2y\cos(y^2)).$$

We can finally obtain the following expression for y':

$$y' = \frac{3x^2y + e^x}{6y - x^3 - 2y\cos(y^2)}.$$