Quiz 3 **Solutions**, Math 111, Section 5 (Vinroot)

1. Compute the derivative of the following function, but do not simplify:

$$g(x) = \frac{x\sin(x)}{x^2 + \cos(x)}$$

Solution: We have to apply the quotient rule, but we also have to apply the product rule for the numerator. First we find the derivative of the numerator using the product rule:

$$\frac{d}{dx}(x\sin(x)) = \frac{d}{dx}(x)\sin(x) + x\frac{d}{dx}(\sin(x)) = 1 \cdot \sin(x) + x\cos(x) = \sin(x) + x\cos(x).$$

Now we can use the quotient rule for the original function. While you were not expected to simplify on the quiz, I will simplify here just to demonstrate how it goes:

$$\begin{aligned}
g'(x) &= \frac{\frac{d}{dx}(x\sin(x))(x^2+\cos(x)) - x\sin(x)\frac{d}{dx}(x^2+\cos(x))}{(x^2+\cos(x))^2} \\
&= \frac{(\sin(x)+x\cos(x))(x^2+\cos(x)) - x\sin(x)(2x-\sin(x))}{(x^2+\cos(x))^2} \qquad \text{(You can stop here on the quiz.)} \\
&= \frac{x^2\sin(x) + x^3\cos(x) + \sin(x)\cos(x) + x\cos^2(x) - 2x^2\sin(x) + x\sin^2(x)}{(x^2+\cos(x))^2} \\
&= \frac{x^3\cos(x) - x^2\sin(x) + \sin(x)\cos(x) + x(\cos^2(x)+\sin^2(x))}{(x^2+\cos(x))^2} \\
&= \frac{x^3\cos(x) - x^2\sin(x) + \sin(x)\cos(x) + x}{(x^2+\cos(x))^2}
\end{aligned}$$

2. Compute the following limit, and make sure all steps are clearly shown or explained.

$$\lim_{x\to 1} \frac{3\sin(x-1)}{x-1}.$$

Solution: First, notice that $x-1$ approaches 0 as x approaches 1. So, if we let $t = x-1$, then x approaching 1 means the same as t approaching 0. Substituting t for $x-1$ in the limit, and substituting $x \to 1$ with $t \to 0$, we have

$$\lim_{x\to 1} \frac{3\sin(x-1)}{x-1} = \lim_{t\to 0} \frac{3\sin(t)}{t}.$$

We have seen that $\lim_{t\to 0} \frac{\sin(t)}{t} = 1$, and so we have

$$\lim_{t\to 0} \frac{3\sin(t)}{t} = 3\lim_{t\to 0} \frac{\sin(t)}{t} = 3 \cdot 1 = 3.$$

Thus, we have

$$\lim_{x\to 1} \frac{3\sin(x-1)}{x-1} = 3.$$