Quiz 1 **Solutions**, Math 111, Section 5 (Vinroot)

Find values of a and b so that the following function f is continuous at all values of x. Be sure to use the meaning of continuity and explain your answer by calculating limits (do **not** use the precise definition of a limit).

$$f(x) = \begin{cases} 2x + 11 & \text{if } x < 0 \\ ax^2 + b & \text{if } 0 \leq x \leq 3 \\ \sqrt{x+1} & \text{if } x > 3. \end{cases}$$

Solution: Note that $2x + 11$ and $ax^2 + b$, being polynomials, are continuous for all values of x, and $\sqrt{x+1}$ is continuous whenever $x \geq -1$, since the square root function is continuous wherever it is defined. This means for $f(x)$ to be continuous, we just need to ensure it is continous at $x = 0$ and $x = 3$, since it is automatically continuous at all other points. This means we need

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0),$$

and we need

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3).$$

Computing, we have

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (2x + 11) = 2(0) + 11 = 11,$$

by continuity of polynomials. Also,

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (ax^2 + b) = a(0^2) + b = b = f(0),$$

again by continuity of polynomials and the definition of $f(0)$. Since these two limits must be equal, we need $b = 11$. Next we compute

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (ax^2 + 11) = a(3^2) + 11 = 9a + 11 = f(3),$$

using continuity of polynomials, the fact that we must have $b = 11$, and from the definition of $f(3)$. Finally, we also can compute

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (\sqrt{x+1}) = \sqrt{3+1} = 2,$$

since $\sqrt{x+1}$ is continuous at $x = 3$. Since these two limits must be equal, then we must have $9a + 11 = 2$, so $a = -1$.

Thus, the two values we need are $a = -1$ and $b = 11$, to guarantee that f is continuous at all values.